8.EE Ant and Elephant

Task

An ant has a mass of approximately 4×10^{-3} grams and an elephant has a mass of approximately 8 metric tons.

a. How many ants does it take to have the same mass as an elephant?

b. An ant is 10^{-1} cm long. If you put all these ants from your answer to part (a) in a line (front to back), how long would the line be? Find two cities in the United States that are a similar distance apart to illustrate this length.

Note: 1 kg = 1000 grams, 1 metric ton = 1000 kg, 1m = 100 cm, 1km = 1000 m

IM Commentary

In this problem students are comparing a very small quantity with a very large quantity using the metric system. The metric system is especially convenient when comparing measurements using scientific notations since different units within the system are related by powers of ten.

Solution

a. First we observe that the mass of the ant is in grams, where the mass of the elephant is in metric tons. We cannot compare the two masses as is, so we first convert them into the same units. We can do this with ratios, using the conversion chart above to convert metric tons into kilograms and then kilograms into grams.

$$(8 \text{ metric tons})\left(\frac{1000 \text{ kg}}{1 \text{ metric ton}}\right)\left(\frac{1000 \text{ grams}}{1 \text{ kg}}\right) = 8,000,000 \text{ grams}$$
$$= 8 \times 10^6 \text{ grams}$$

Notice that both metric tons and kg appear once on both the top and the bottom, enabling them to be canceled out, leaving only grams. Also note that we could have converted .003 grams into metric tons, and though this would have given an answer with a large number of decimal places, we would end up with the same final result.

Now that both quantities are in grams, we want to find how many ants, n, are required to have the same mass as the elephant. As each ant has a mass of 4×10^{-3} grams, we multiply 4×10^{-3} by n to get the total mass of the ants

$$\text{total mass of } n \text{ ants} = 4 \times 10^{-3} n$$

and then we set this equal to the mass of the elephant:

$$4 \times 10^{-3} n = 8 \times 10^6$$

We find that $n = 2 \times 10^9$ ants.

Thus, it takes approximately 2×10^9 ants to have the same mass as an elephant.

b. Each ant is 10^{-1} cm long, and we have 2×10^9 ants total. If we put all these ants in a line (front to back), we can find the length of the line by multiplying the length of one ant by how many ants we have:

$$2 \times 10^9 \times 10^{-1} \text{ cm} = 2 \times 10^8 \text{ cm}$$

As is, this answer is difficult to illustrate because it is such a large number. To find two objects that are this distance apart, we first want to convert centimeters into more useful units. We do this by using ratios from the above chart, converting centimeters to meters and meters to kilometers.

$$(2 \times 10^8 \text{ cm})\left(\frac{1 \text{ m}}{10^2 \text{ cm}}\right)\left(\frac{1 \text{ km}}{10^3 \text{ m}}\right) = 2 \times 10^3 \text{ km} = 2000 \text{ km}$$

This amount is much easier to visualize, and it turns out that 2000 km is approximately the driving distance from San Francisco, California to Denver, Colorado.

Illustrative
Mathematics