

8.EE Fixing the Furnace

Task

Ivan's furnace has quit working during the coldest part of the year, and he is eager to get it fixed. He decides to call some mechanics and furnace specialists to see what it might cost him to have the furnace fixed. Since he is unsure of the parts he needs, he decides to compare the costs based only on service fees and labor costs. Shown below are the price estimates for labor that were given to him by three different companies. Each company has given the same time estimate for fixing the furnace.

- Company A charges \$35 per hour to its customers.
- Company B charges a \$20 service fee for coming out to the house and then \$25 per hour for each additional hour.
- Company C charges a \$45 service fee for coming out to the house and then \$20 per hour for each additional hour.

For which time intervals should Ivan choose Company A, Company B, Company C? Support your decision with sound reasoning and representations. Consider including equations, tables, and graphs.

IM Commentary

This task can be used to both assess student understanding of systems of linear equations or to promote discussion and student thinking that would allow for a stronger solidification of these concepts. The solution can be determined in multiple ways, including either a graphical or algebraic approach.

Modifications of the task might have students only focus on two of the companies at a time. Another modification might be to adjust the values of the service fee and hourly rate so that some of them match.

There are also many extensions or variations of the task that can be useful when working with students. For example, the teacher can ask the students, “What adjustment need to be made so that all three graphs will intersect at one point?” This requires them to understand how to shift one of the graphs so they all intersect at the same point. Here are some questions to facilitate the work of students as they engage with the task or as part of a whole-group discussion:

This task was submitted by Travis Lemon for the second Illustrative Mathematics task writing contest 2011/01/09.

- How can we create a price plan that has a service fee but is always \$15 less than company C?
- How can we make company A more competitive for a larger number of hours of labor?
- What if a fourth company decides to price match labor costs of their competitors? How would their pricing look?

The Standards for Mathematical Practice focus on the nature of the learning experiences by attending to the thinking processes and habits of mind that students need to develop in order to attain a deep and flexible understanding of mathematics. Certain tasks lend themselves to the demonstration of specific practices by students. The practices that are observable during exploration of a task depend on how instruction unfolds in the classroom. While it is possible that tasks may be connected to several practices, only one practice connection will be discussed in depth. Possible secondary practice connections may be discussed but not in the same degree of detail.

This task helps illustrate Mathematical Practice Standard 1, “Make sense of problems and persevere in solving them.” Students are given multiple methods for solving the problem, and they can choose which makes the most sense to them. Students will need to decide on an entry point and mathematical model, either a graph, table, or equations. Once they begin to work on the task, because of the three different situations to compare, they will need to continually ask themselves, “Does this make sense?” Finally, because this task goes beyond an average comparison of two different companies, students might need to persevere in their solving of the task, and know what types of assumptions make a difference in the solution.

Solution

Let x be the number of hours it takes to fix the furnace, and y the cost in dollars of fixing the furnace. Company A's cost can be modeled with the equation

$$y = 35x,$$

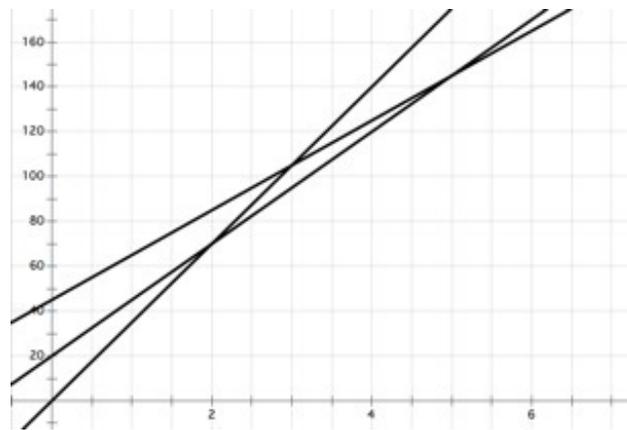
company B's with the equation

$$y = 25x + 20,$$

and company C's with the equation

$$y = 20x + 45.$$

Graphing the equations helps students visualize the solution.



Looking at the graph, it appears that the graph that represents the cost for company A is below the other two graphs for any amount of time less than 2 hours, where the graph intersects the line that represents the cost of company B. The graph for company B is below the other two up until 5 hours, where it intersects the graph that represents the cost for company C. In other words, company A is least expensive for 0 to 2 hours, company B is least expensive for 2 to 5 hours, and company C is least expensive for more than five hours.

Students should get in the habit of checking the coordinates of the intersection points algebraically, since it is often the case that the coordinates of the intersection points are not whole numbers (and therefore not easy to read from the graph). This also reinforces the relationship between the algebraic representation of the solution and the graphical representation of the solution.

To find the solution algebraically, consider each pair of equations as a system. Let x be the number of hours it takes to repair the furnace and y be the cost of the repair (without parts).

To find the number of hours for which company A and company B cost the same, consider $y = 35x$ and $y = 25x + 20$. Substituting for y , we get

$$35x = 25x + 20.$$

The solution to this equation gives the number of hours for which company A and company B cost the same. Solving this equation, we find that the cost of company A and company B is the same for 2 hours of labor. The cost is \$70.

To find the number of hours for which company A and company C cost the same, consider $y = 35x$ and $y = 20x + 45$. Substituting for y , we get

$$35x = 20x + 45.$$

Solving this equation, we find that the cost of company A and company C is the same for 3 hours labor, for a total cost of \$105.

To find the number of hours for which company B and company C cost the same, consider $y = 25x + 20$ and $y = 20x + 45$. Substituting for y , we get

$$25x + 20 = 20x + 45.$$

Solving this equation, we find that the cost of company B and company C is the same for 5 hours labor, for a total cost of \$145.

Some additional substitution of values shows that company C is \$15 more expensive than A and B at 2 hours. Company B is \$10 less than A and C at 3 hours. And company A is \$30 more expensive at 5 hours than companies B and C.

As before, we found that company A is the least expensive up to a time of 2 hours, at which point company A and B are the same cost. From 2 hours to 5 hours, company B is the least expensive, and at 5 hours company B and C both cost \$145. For more than 5 hours, Company C will be the least expensive.



