8.EE Cell Phone Plans

Task

You are a representative for a cell phone company and it is your job to promote different cell phone plans.

a. Your boss asks you to visually display three plans and compare them so you can point out the advantages of each plan to your customers.

- Plan A costs a basic fee of $29.95 per month and 10 cents per text message
- Plan B costs a basic fee of $90.20 per month and has unlimited text messages
- Plan C costs a basic fee of $49.95 per month and 5 cents per text message
- All plans offer unlimited calling
- Calling on nights and weekends are free
- Long distance calls are included

b. A customer wants to know how to decide which plan will save her the most money. Determine which plan has the lowest cost given the number of text messages a customer is likely to send.

IM Commentary

This task presents a real-world problem requiring the students to write linear equations to model different cell phone plans. Looking at the graphs of the lines in the context of the cell phone plans allows the students to connect the meaning of the intersection points of two lines with the simultaneous solution of two linear equations. The students are required to find the solution algebraically to complete the task. Note that the last three pieces of information describing the plans are superfluous; it is important for students to be able to sort through information and decide what is, and is not, relevant to solving the problem at hand.

This task was submitted by James E. Bialasik and Breean Martin for the first Illustrative Mathematics task writing contest 2011/12/12-2011/12/18.

Solution

a. All three plans start with a basic monthly fee; in addition, the costs for Plans A and C increase at a steady rate based on the number of text messages sent per month. Therefore, we can find a linear equation for each plan relating y, the total monthly cost in dollars, to t, the number of text messages sent.

Plan A has a basic fee of \$29.95 even if no text messages are sent. In addition, each text message costs 10 cent or \$0.10. We can write the total cost per month as

$$y = 29.95 + 0.10t$$

Plan B has a basic fee of \$90.20 even if no text messages are sent. In this case the total cost per month, y, does not change for different values of t, so we have

$$y = 90.20$$

Plan C has a basic fee of \$49.95 even if no text messages are sent. In addition, each text message costs 5 cent or \$0.05. We can write the total cost per month as

$$y = 49.95 + 0.05t$$

To visually compare the three plans, we graph the three linear equations. In each case the basic fee is the vertical intercept, since it indicates the cost of a plan even if no text messages are being sent. The graph for the Plan B equation is a constant line at $y = 90.20$. Plan B has a lower basic fee (\$29.95) than Plan A (\$49.95); therefore it starts lower on the vertical axis. Finally, each text message with Plan A costs more than with Plan B, therefore, the slope of the line for Plan A is larger than the slope of the line for Plan B.
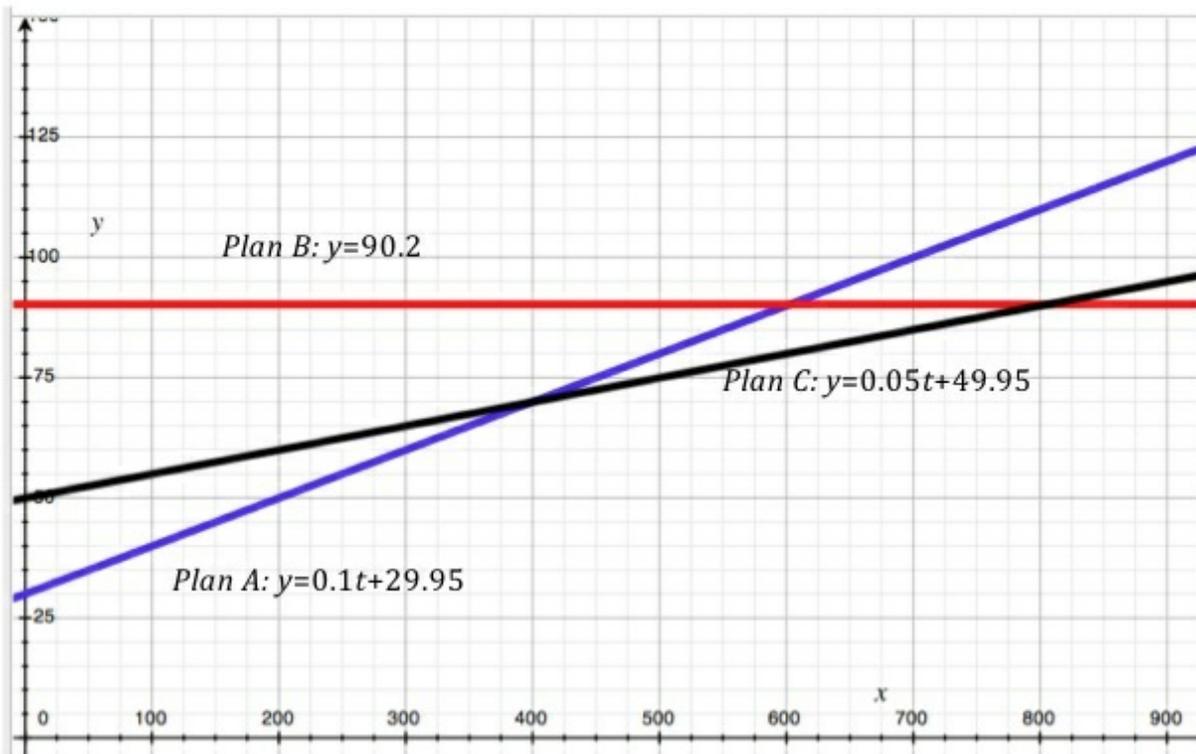
From the graphical representation we see that the "best" plan will vary based on the number of text messages a person will send. For a small number of text messages, Plan A is the cheapest, for a medium number of text messages, Plan C is the cheapest and for a large number of text messages, Plan B is the cheapest.

At an intersection point of two lines, the two plans charge the same amount for the same number of text messages. To determine the range of "small", "medium" and "large" numbers of text messages, we need to find the t-coordinate of the intersection points of the graphs. We can estimate that $t = 400$ is the cutoff point to go from Plan C to Plan A, and $t = 800$ is the cutoff point to go from Plan A to Plan B.

b. To find the exact coordinates of each intersection point, we need to solve the corresponding system of equations. The coordinates of these points correspond to the exact number of text messages for which two plans charge the same amount. Because we are looking for the number of text messages, t, that result in the same cost for two different plans, we can set the expression that represents the cost of one plan equal to the other and solve for t.

Plan A = Plan C

$$0.1t + 29.95 = .05t + 49.95$$
$$.05t = 20$$
$$t = 400 \quad \text{Text Messages}$$

Plan C = Plan B

$$0.05t + 49.95 = 90.20$$
$$0.05t = 40.25$$
$$t = 805 \quad \text{Text Messages}$$

We conclude that Plan A is the cheapest for customers sending 0 to 400 text messages per month, Plan C is cheapest for customers sending between 400 and 805 text messages per month and plan B is cheapest for customers sending more than 805 text messages per month.