# 7.RP Comparing Years

## Task

Historically, different people have defined a year in different ways. For example, an Egyptian year is 365 days long, a Julian year is $365\frac{1}{4}$ days long, and a Gregorian year is 365.2425 days long.

a. What is the difference, in seconds, between a Gregorian year and a Julian year?

b. What is the percent decrease, to the nearest thousandth of a percent, from a Julian year to a Gregorian year?

c. How many fewer days are there in 400 years of the Gregorian calendar than there are in 400 years of the Julian calendar?

## IM Commentary

Many students will not know that when comparing two quantities, the percent decrease between the larger and smaller value is not equal to the percent increase between the smaller and larger value. Students would benefit from exploring this phenomenon with a problem that uses smaller values before working on this one.

## Solutions

### Solution: Find the number of seconds in a year first and then subtract

a. To find how many seconds in a day, multiply the number of seconds in a minute by the number of minutes in an hour by the number of hours in a day:

$$\frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{24 \text{ hours}}{1 \text{ day}} = \frac{86,400 \text{ seconds}}{1 \text{ day}}$$

To find the number of seconds in a year, multiply the length of the year in days by the number of seconds in a day.

- For a Julian year:

$$\frac{365.25 \text{ days}}{1 \text{ year}} \times \frac{86,400 \text{ seconds}}{1 \text{ day}} = \frac{31,557,600 \text{ seconds}}{1 \text{ year}}$$

- For a Gregorian year:

$$\frac{365.2425 \text{ days}}{1 \text{ year}} \times \frac{86,400 \text{ seconds}}{1 \text{ day}} = \frac{31,556,952 \text{ seconds}}{1 \text{ year}}$$

Now subtract the length of a Gregorian year from the length of a Julian year:

$$31,557,600 - 31,556,952 = 648$$

So a Julian year is 648 seconds longer than a Gregorian year.

b. To find how much shorter a Gregorian year is than a Julian year (as a percentage of a Julian year), divide the difference by the length of a Julian year:

$$648 \div 31557600 = 0.00002053...$$

which is approximately $\frac{2}{100,000}$ or 0.002%.

So a Gregorian year is about 0.002% shorter than a Julian year.

c. Multiplying 365.25 by 400, we get 146,100 days for the Julian calendar. Multiplying 365.2425 by 400, we get 146,097 days for the Gregorian calendar. We see a decrease of 3 days in 400 years. If we round 146,100 days to 150,000 days, we see that 3 units less in 150,000 units is consistent with our earlier finding of approximately 1 unit less per 50,000 units.

**Solution: Find the difference in the number of days first and then convert to seconds**

a. We can subtract the length of a Gregorian year from a Julian year:

$$365.25 - 365.2425 = 0.0075$$

So there are 0.0075 more days in the Julian year. To convert the number of days to the number of seconds, we can multiply number of seconds in one day with the number of days:

$$86,400 \times 0.0075 = 648$$

So there are 648 more seconds in the Julian year than in the Gregorian year.

b. To find how much shorter a Gregorian year is than a Julian year (as a percentage of a Julian year), divide the difference by the length of a Julian year:

$$648 \div 31557600 = 0.00002053...$$

which is 0.002053... %.

Rounding, we see that a Gregorian year is about 0.002% shorter than a Julian year.

c.

d. Multiplying 365.25 by 400, we get 146,100 days for the Julian calendar. Multiplying 365.2425 by 400, we get 146,097 days for the Gregorian calendar. We see a decrease of 3 days in 400 years.