# A2.7 Making Inferences

**• Understand that statistical methods are used to draw conclusions from data.**
**• Understand that the validity of data-based conclusions depends on the quality of the data and how the data were collected.**
**• Critique and evaluate data-based claims that appear in popular media.**
**• Distinguish between observational studies, surveys and experiments.**
**• Explain why random selection is important in the design of observational studies and surveys.**
**• Explain why random assignment is important in the design of statistical experiments.**
**• Calculate and interpret the standard deviation as a measure of variability.**
**• Use the normal distribution as a model for data distributions that are approximately symmetric and bell-shaped.**
**• Use the least squares regression line to model linear relationships in bivariate numerical data.**
**• Understand sampling variability in the context of estimating a population or a population mean.**
**• Use data from a random sample to estimate a population proportion.**
**• Use data from a random sample to estimate a population mean.**
**• Calculate and interpret margin of error in context.**
**• Understand the relationship between sample size and margin of error.**
**• Given data from a statistical experiment, create a randomization distribution.**
**• Use a randomization distribution to determine if there is a significant difference between two experimental conditions.**

This unit builds on the foundation of One Variable Statistics and Bivariate Statistics, as well as students' work with statistics in grade 7. In particular, the concepts of sampling variability and distributions introduced in earlier units are critical to understanding the process of drawing conclusions from data, which is central to this unit.

In this unit, students revisit and extend their understanding of variability in data and of ways to describe variability in data. Students calculate and interpret the standard deviation, first introduced at a conceptual level in One Variable Statistics, as a way to quantify variability.

Students use distributions to describe variability. Normal distributions are introduced as a way to model data distributions that are bell-shaped and approximately symmetric. Students calculate and interpret areas under a normal curve in the context of modeling a data distribution.

Students explore the reasoning that allows them to draw conclusions based on data from statistical studies. They learn the distinction between an observational study and a statistical experiment. They then use data from a random sample from a population to estimate the values of population characteristics such as a population mean or a population proportion. Students develop the understanding that such estimates are subject to sampling variability. The notion of margin of error is introduced as a way of quantifying the uncertainty associated with an estimate of a population characteristic.

Students also explore the important idea of “statistical significance” as they use data from statistical experiments to determine if there is a significant difference between experimental conditions.

The standards addressed in this unit (especially S-IC.B.4 and S-IC.B.5) are conceptually complex and will require several weeks to fully develop.

Together with One Variable Statistics, Bivariate Statistics, and Probability, this unit provides a capstone experience in statistics for grades 6–12 and a solid foundation for an AP Statistics course or a college level introductory statistics course.

## A2.7.0 Pre-unit diagnostic assessment

**Assess students’ ability to**
- **construct and interpret a graphical display;**
- **calculate and interpret a sample proportion;**
- **calculate and interpret a sample mean;**
- **fit a line to bivariate data.**

## A2.7.1 Drawing reasonable conclusions

- **Understand that statistical methods are used to draw conclusions from data.**

**• Understand that the validity of data-based conclusions depends on the quality of the data and how the data were collected.**
**• Critique and evaluate data-based claims that appear in popular media.**

Pick up any newspaper or magazine and you are likely to see statements that claim to be based on data. But were the data collected in a reasonable way? Are the conclusions drawn from the data reasonable and if so, to whom do they apply?

The goal of this section is to get students to think about conclusions based on data and to begin to think about aspects of study design that will be studied in more detail in the later sections of this unit. In this section, you can present students with statements for a recent newspaper or magazine, or you can use some of the ones that follow here. Encourage students to think about what data were collected and how the data were collected. Ask students if they have any reason to question the statement that was made. What additional information would they want to know before making a decision about the reasonableness of the claim?

You can ask students to bring in a headline or data-based claim that they find in a newspaper or online and use them as the basis for discussion on the second day of this section.

Sample data-based statements for discussion:

- *Women's World* (September 27, 2010): Eating cheese before going to bed will help you sleep better. Eating garlic prevents colds.
- *Associated Press* (September 1, 2002): Vitamins found to prevent blocked arteries.
- *Women's World* (November 1, 2010): Strengthen your marriage with prayer.
- *Food Network Magazine* (January 2012): People who push a shopping cart at a grocery store are less likely to purchase junk food that those who use a hand-held basket.
- *The Los Angeles Times* (September 25, 2009): Spanking lowers a child's IQ.

The following article might also be of interest: "Health Freaks on Trial: Duct Tape, Bull Semen and the Call of Television," *Significance*, April 2014, Volume 11, Issue 2.

## A2.7.2 Collecting data and types of statistical studies

**• Distinguish between observational studies, surveys and experiments.**

**• Explain why random selection is important in the design of observational studies and surveys.**
**• Explain why random assignment is important in the design of statistical experiments.**

Data are usually collected in order to answer some question. Depending on the nature of the question, data collection usually involves either observing characteristics of a sample from some population or carrying out a statistical experiment. This section introduces three types of statistical studies: observational studies, surveys, and experiments.

Observational studies and surveys are designed to answer questions about a population, and usually involve generalizing from a sample to a larger population of interest. This means that it is important that the sample be selected in a way that is likely to produce data representative of the population. Remind students of their prior work with random sampling in grade 7, and if necessary, review methods for selecting a random sample and discuss why random selection is important in the design of observational studies and surveys.

Statistical experiments are usually designed to answer questions of the form "What happens if...?" or "What is the effect of ...?" Provide examples of questions that could be answered by using data from a statistical experiment (such as "What is the effect of listening to music while studying on exam performance?" or "What happens to the moisture content of tortilla chips if the frying time is increased from 15 seconds to 20 seconds?")

Provide examples of statistical experiments that compare two or more experimental conditions. In a statistical experiment, it is important to start with comparable experimental groups. Discuss how random assignment of subjects to experimental groups is one way to ensure this.

Emphasize the difference between random selection and random assignment, and focus on the purpose of random selection and of random assignment in study design.

Provide examples of each type of study and allow students to practice distinguishing between the different types of studies. You might also ask students to bring in study descriptions from the newspaper or other media sources and have them explain to the class what type of study is being described and whether it involves random selection or random assignment.

**Tasks**
S-IC Why Randomize?
S-IC Words and Music II
S-IC Strict Parents

## A2.7.3 Describing data distributions

**• Calculate and interpret the standard deviation as a measure of variability.**
**• Use the normal distribution as a model for data distributions that are approximately symmetric and bell-shaped.**
**• Use the least squares regression line to model linear relationships in bivariate numerical data.**

This section reviews material from grades 6–8 and previous units and forms a bridge between the previous section on collecting data and later sections on drawing conclusions from data.

The focus of this section is on three main ideas: (1) quantifying variability in a data set, (2) the normal distribution as a model for data distributions, and (3) the least squares regression line as a summary of a bivariate linear relationship. (Note that students are not required to know the term “least squares regression line,” referring to it instead as “line of best fit.”)

In the unit One Variable Statistics, students were introduced to the idea of standard deviation as a measure of variability. The focus in this unit should be on calculating the standard deviation (using technology) and on interpreting the standard deviation in context.

Students will probably not have seen the normal distribution before this unit, so it will need to be introduced here. Focus on properties of the normal distribution. Students will need to be able to find areas under a normal curve and percentiles for a normal distribution. The use of technology to assist in these calculations is encouraged. Students should be asked to interpret areas under a normal curve and percentiles for a normal distribution in a variety of different contexts.

In the unit Bivariate Statistics, students used technology to find the line of best fit (least squares regression line). This is revisited here with a focus on interpretation and modeling.

## A2.7.4 Mid-unit assessment

**Assess students' ability to**
**• given a description of a statistical study, identify the study type (observational study, survey, or experiment);**
**• determine what type of statistical study would produce data that could be used to answer a given question;**
**• distinguish between data distributions for which it would be reasonable to use the normal distribution as a model and those for which it would not be reasonable;**
**• find an area under a normal curve and interpret it in the context of modeling a data distribution.**

## A2.7.5 Drawing conclusions based on data from a random sample

**• Understand sampling variability in the context of estimating a population or a population mean.**
**• Use data from a random sample to estimate a population proportion.**
**• Use data from a random sample to estimate a population mean.**
**• Calculate and interpret margin of error in context.**
**• Understand the relationship between sample size and margin of error.**

A good description of how this section might play out in the classroom can be found on pp. 8–10 of Progressions for the Common Core State Standards in Mathematics: High School Statistics and Probability.

The focus of this section is on standards S-IC.B.3 and S-IC.B.4. This section will require a substantial time commitment. Students build on what they have learned about distributions and sampling variability as they use data from a random sample to learn about the value of a population proportion or a population mean.

It is probably easiest to begin by focusing on estimating a population proportion. Simulation can be used to approximate the sampling distribution of a sample proportion. This can be done using physical simulations (for example from a bag of beads that contains 40% blue beads) or using one of the many technology applets (for example, rossmanchance.com/applets/Reeses3/ReesesPieces.html which builds up a sampling distribution of the proportion of orange candies in a random sample after the user specifies the population proportion and the sample size). It is recommended that at least one physical simulation be carried out before turning to technology so that students understand the process that the technology is implementing.

Simulated sampling distributions become the basis for the important discussion of how the sampling distribution provides information about the anticipated accuracy of estimates based on random samples.

Margin of error associated with an estimate of a population proportion can be motivated based on the simulated sampling distribution in one of two ways:

1. In the context of the sampling distribution of a sample proportion, general properties can be described. These include:

- The sampling distribution is approximately normal if the sample size is large enough and the population proportion is not too close to 0 or 1.
- The sampling distribution is centered at the value of the population proportion, meaning that sample proportions tend to cluster around the actual value of the population proportion.
- The standard deviation of the sampling distribution (the standard deviation of the sample proportion) is approximately equal to $\sqrt{p(1-p)/n}$.

Based on these properties, the margin of error for estimating a population proportion is approximately $2\sqrt{p(1-p)/n}$.

2. A less formal approach introduces $1/\sqrt{n}$ as a conservative estimate of the margin of error (this is the maximum value of $2\sqrt{p(1-p)/n}$, which occurs when $p = ½$. The simulated sampling distributions can then be used to convince students that this value is reasonable (and at times a bit large—this is the "conservative" part) as a way to describe error.

Once margin of error has been developed, the formula and/or simulation results can be used to explore the relationship between sample size and margin of error.

Students should practice interpreting margin of error in context. This is a good place to bring in statements from the media where a margin of error is reported and to have students explain what is meant by the margin of error.

The sampling distribution also provides information that allows testing of claims about a population proportion. By comparing an observed sample proportion to what would be expected under a specified model (for example, a model that specifies that the population proportion is 0.6), a decision can be reached about whether the observed data are consistent with the model or whether it provides evidence that the model is not a believable description of the population.

The final part of this section should consider using sample data to estimate a population mean. Simulating to obtain a sampling distribution is not as easy here as in the case of proportions, but there are a number of good applets that can be used to carry out such simulations, e.g., rossmanchance.com/applets/SampleMeans/SampleMeans.html).

**Tasks**
S-IC Sarah, the chimpanzee
S-IC Block Scheduling

## A2.7.6 Drawing conclusions based on data from a statistical experiment

**• Given data from a statistical experiment, create a randomization distribution.**
**• Use a randomization distribution to determine if there is a significant difference between two experimental conditions.**

A good description of how this section might play out in the classroom can be found on pp. 10–12 of Progressions for the Common Core State Standards in Mathematics: High School Statistics and Probability.

This section focuses on using data from statistical experiments to determine if there is a significant difference between two experimental conditions. By investigating what group differences might be expected due to chance alone when subjects are randomly partitioned into two groups, students are able to determine if chance alone is a plausible explanation for an observed difference. It is important that students understand that when a single group is partitioned into two groups, the two groups will tend to differ just by chance. This idea is fundamental in distinguishing "significant differences" from differences that might be due only to chance.

In this section, students should use data from an experiment to create a randomization distribution as a way of exploring group differences that are consistent with chance. This distribution is then used to determine if an observed difference is consistent with chance or whether the difference is large enough to indicate a significance difference due to the effects of the experimental conditions. Both physical and technology assisted simulations can be used to develop randomization distributions.

## A2.7.7 Summative assessment

**Assess students' ability to**
**• distinguish between an observational study and an experiment;**
**• use an appropriate normal distribution to model a data distribution;**
**• estimate a population proportion and interpret a margin of error in context;**
**• given a simulated sampling distribution, estimate a margin of error;**
**• given data from a statistical experiment, create a randomization distribution and use it to determine if there is a significant difference between experimental conditions.**