## M3.4 Circles

- Use the Pythagorean Theorem to derive an equation for a circle of given center and radius.
- Use similarity to derive the fact that the length of the arc of a circle intercepted by an angle is proportional to the radius of the circle.
- Derive a formula for the area of a sector.
- Identify and describe relationships between central and inscribed angles and their arcs.
- Prove that an inscribed angle that subtends a diameter is a right angle, and its converse.
- Identify and describe relationships and ratios of lengths for intersecting chords.
- Prove that a radius and a tangent to a circle at the same point are perpendicular.
- Prove properties of angles of inscribed polygons.
- Use relationships about inscribed angles to solve problems about inscribed polygons.
- Use circles, cones, tangent segments, chords, and related figures, and their properties to describe objects.

Students begin this unit by working with circles on the coordinate plane and use the Pythagorean Theorem to derive the equation of a circle. They learn about radian measure and use similarity to derive the fact that the length of the arc intercepted by an angle is proportional to the radius of the circle. They develop a formula for the area of a sector of a circle and examine properties of central and inscribed angles in a circle along with their subtended arcs. Students also examine properties of tangent lines, radii, and intersecting chords and apply these properties to solve problems in a variety of contexts, including real-world situations.

In grade 4, students learned to view an angle as indicating an amount of "turn," e.g., "An angle that turns through 1/360 of a circle is called a "one-degree angle,' and can be used to measure angles."4.MD. 5 In unit G1, students established a precise definition for a circle, e.g., that a circle is the locus of all points at a given distance from a given point. They made straightedge and compass constructions of circles, perpendicular bisectors, angle bisectors, and midpoints. From unit G3, they know that all circles are similar and they have gained experience using triangle congruence criteria. In unit G4, students were introduced to right triangle trigonometry. Unit G5 concerned solid geometry, including one task (Use Cavalieri's Principle to Compare Aquarium Volumes) that involved working with a cross-section of a sphere. Students begin this unit by working with circles on the coordinate plane, and use the Pythagorean Theorem to derive the equation of a circle. They learn about radian measure and a formula for the area of a sector of a circle. They examine properties of central and inscribed angles in a circle, and their subtended arcs. Students examine properties of tangent lines and radii, and intersecting chords. They apply these properties in a variety of contexts, including real-world contexts. After this unit, students extend the domain of trigonometric functions to the unit circle in the coordinate plane, working with radian measure, and again making use of the Pythagorean Theorem.

## M3.4.0 Diagnostic pre-unit assessment

## Diagnose students' ability to

- find the area of a circle and of a sector of a circle;
- find measure of a sector angle when the sector partitions the circle into congruent pieces;
- construct a perpendicular bisector;
- find the distance between two given points on the coordinate plane.

Students begin this unit by working with circles on the coordinate plane and use the Pythagorean Theorem to derive the equation of a circle. They learn about radian measure and use similarity to derive the fact that the length of the arc intercepted by an angle is proportional to the radius of the circle. They develop a formula for the area of a sector of a circle and examine properties of central and inscribed angles in a circle along with their
subtended arcs. Students also examine properties of tangent lines, radii, and intersecting chords and apply these properties to solve problems in a variety of contexts, including real-world situations.

## M3.4.1 Equations of circles

## Use the Pythagorean Theorem to derive a general equation for a circle of given center and radius.

Students begin this unit by working with circles on the coordinate plane and use the Pythagorean Theorem to derive the equation of a circle. They learn about radian measure and use similarity to derive the fact that the length of the arc intercepted by an angle is proportional to the radius of the circle. They develop a formula for the area of a sector of a circle and examine properties of central and inscribed angles in a circle along with their subtended arcs. Students also examine properties of tangent lines, radii, and intersecting chords and apply these properties to solve problems in a variety of contexts, including real-world situations.

## M3.4.2 Arc lengths, sectors, and radians

## - Use similarity to derive the fact that length of an intercepted arc of an angle is proportional to the length of the radius of the circle. <br> - Define radian measure of an angle. <br> - Derive a formula for the area of a sector. <br> - Use the formula in solving problems.

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## M3.4.3 Central and inscribed angles

- Identify and describe relationships between central and inscribed angles and their arcs.
- Prove that an inscribed angle that subtends a diameter is a right angle, and, conversely, that a right angle inscribed in a circle must subtend a diameter.
- Apply these relationships in various contexts.

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## M3.4.4 Mid-Unit Assessment

## Assess students' ability to

- solve area problems involving sectors;
- use properties of central angles and inscribed angles to find arc length.

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## M3.4.5 Radii, tangent lines, chords, and secants

- Prove that a radius and a tangent line that intersect at the same point are perpendicular.
- Identify and describe relationships and ratios of lengths for intersecting chords.

Students begin this unit by working with circles on the coordinate plane and use the Pythagorean Theorem to derive the equation of a circle. They learn about radian measure and use similarity to derive the fact that the length of the arc intercepted by an angle is proportional to the radius of the circle. They develop a formula for the area of a sector of a circle and examine properties of central and inscribed angles in a circle along with their subtended arcs. Students also examine properties of tangent lines, radii, and intersecting chords and apply these properties to solve problems in a variety of contexts, including real-world situations.

## M3.4.6 Inscribed and circumscribed polygons

## - Prove that the opposite angles in a cyclic quadrilateral that contains the center of the circle are supplementary.

- Construct the inscribed and circumscribed circles of a triangle.

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## M3.4.7 Applications

Use circles, cones, tangent segments, chords, and related figures, and their

## properties to describe objects.

Students begin this unit by working with circles on the coordinate plane and use the Pythagorean Theorem to derive the equation of a circle. They learn about radian measure and use similarity to derive the fact that the length of the arc intercepted by an angle is proportional to the radius of the circle. They develop a formula for the area of a sector of a circle and examine properties of central and inscribed angles in a circle along with their subtended arcs. Students also examine properties of tangent lines, radii, and intersecting chords and apply these properties to solve problems in a variety of contexts, including real-world situations.

## M3.4.8 Summative Assessment

## Assess students' ability to apply knowledge of circle properties.

Students begin this unit by working with circles on the coordinate plane and use the Pythagorean Theorem to derive the equation of a circle. They learn about radian measure and use similarity to derive the fact that the length of the arc intercepted by an angle is proportional to the radius of the circle. They develop a formula for the area of a sector of a circle and examine properties of central and inscribed angles in a circle along with their subtended arcs. Students also examine properties of tangent lines, radii, and intersecting chords and apply these properties to solve problems in a variety of contexts, including real-world situations.

