# M3.1 Exponential Functions 2

**• Create and analyze a simple exponential function arising from a real-world or mathematical context.**
**• Evaluate and interpret exponential functions at non-integer inputs.**
**• Understand functions of the form $f(t) = P(1 + r/n)^{nt}$ and solve problems with different compounding intervals.**
**• Understand informally how the base $e$ is used in functions to model a quantity that compounds continuously.**
**• Write exponential expressions in different forms.**
**• Explain what the parameters of an exponential function mean in different contexts.**
**• Use the properties of exponents to write expressions in equivalent forms.**
**• Build exponential functions to model real world contexts.**
**• Analyze situations that involve geometric sequences and series.**
**• Derive the formula for the sum of a finite geometric series.**

In previous units students have worked with geometric sequences and understand they change by a constant ratio over a constant interval. They are able to write both recursive and closed equations for them. Students understand the difference between a linear and exponential function, can recognize situations and tables described by each, and know that an exponential function will always overtake a linear function. They know that an exponential function grows increasingly rapidly in one direction, and approaches a value asymptotically in the other direction. They have solved exponential equations of the form abx = c by graphing. They can construct an exponential function given a graph, description of a relationship, or two input output pairs with integer inputs (including in a table). Given an expression defining an exponential function, they can interpret its parameters in a context. They can also fit a simple exponential function to a scatterplot. Every exponential function until now has only involved integer inputs.

In this unit students broaden their view of exponential functions to include the entire real number line as a possible domain. They learn about functions with base $e$. An approach using compound interest that shows e arising as the natural base for a quantity being compounded continuously can serve as a way to develop understanding appropriate to this level. First, students must understand functions of the form $f(x) = P(1 + r/n)^{nt}$ which show a given compounding frequency, $n$.

Students examine some different forms of exponential functions and learn to interpret the parameters in terms of a context. They learn the concept of doubling time and see functions expressed in a form that shows the doubling time; they work algebraically with functions expressed in a form like $f(x) = A(1 + r/n)^{nt}$ that shows the compounding period; and they work with functions written with the base e, $g(x) = Ae^{rt}$, in many continuous growth contexts. Students build functions in those forms in order model real-world contexts. Contexts may include Moore's law for computer processor speeds, population growth, and temperature change. Students should also consider whether an exponential or a linear model is appropriate in various contexts.

Students analyze situations that involve summing an exponential sequence (which is generally called a geometric series) These arise naturally in some saving and banking problems as well as in some interesting geometric contexts. Students will build on previous work with geometric sequences to derive a formula for the sum of a geometric series.

A later unit introduces logarithms as the solutions to exponential equations. Ultimately, students are proficient at graphing, analyzing, solving, and modeling exponential situations. Students are comfortable with exponentials from a functions viewpoint. This lays the foundations for success in calculus.

## M3.1.0 Pre-unit diagnostic assessment

## M3.1.1 Understanding exponential growth and decay

**Create and analyze a simple exponential function arising from a real-world or mathematical context.**

Students have been introduced to exponential functions in A2. For some students this introduction could have occurred a year or two previously.

Therefore this unit starts with an activity that re-introduces exponential growth and decay in an engaging real-world or mathematical context. The context can be modeled by an exponential function with domain contained in the integers, thus providing a review of previous experience as needed for students.

## M3.1.2 Exponential functions on the real numbers

**Evaluate and interpret exponential functions at non-integer inputs.**

In A2 students considered exponential functions at integer inputs only. Now that they understand how to determine the value of $b^x$ for any rational number $x$, they can approximate $b^x$ to any degree of accuracy for any real number $x$. In this section they broaden their view of exponential functions to include the entire real number line as a possible domain. They evaluate functions and interpret their values at real inputs in terms of a context, in preparation for the more sophisticated work in the following sections.

## M3.1.3 Changing compounding intervals, continuous compounding, and the base e

- **Understand functions of the form $f(t) = P(1 + r/n)^{nt}$ which use a given compounding frequency, $n$.**
- **Solve problems given different compounding intervals.**
- **Understand (informally) how the base $e$ arises in the context of compounding intervals as $n$ becomes arbitrarily large.**

The base $e$ is very commonly used in scientific and other modeling applications. The fundamental reason that $e$ is a useful base for an exponential function is beyond the scope of this course. However, an approach using compound interest that shows $e$ arising as the natural base for a quantity being compounded continuously can serve as a way to develop understanding at this level. First, students must understand functions of the form $f(t) = P(1 + r/n)^{nt}$ which show a given compounding interval, $n$. This section examines that approach.

**Tasks**

A-SSE The Bank Account
F-BF Compounding with a 5% Interest Rate

## M3.1.4 Interpreting exponential functions

**• Solve problems involving exponential functions in many different contexts.**
**• Write exponential expressions in different forms.**
**• Explain what the parameters of an exponential function mean in different contexts.**
**• Use the properties of exponents to write expressions in equivalent forms.**

In Unit A2, students worked with exponential functions in the form $f(t) = ab^t$ or $f(t) = a(1 + r)^t$ and interpreted the parameters $a$, $b$, and $r$ in terms of a context. In this unit they see more complicated forms. In the previous section, students developed an understanding of different compounding intervals and continuous compounding using base $e$. The purpose of this section is to examine some of these different forms and learn to interpret the parameters in terms of a context. Students learn the concept of doubling time and see functions expressed in a form that shows the doubling time; they work algebraically with functions expressed in a form like $f(x) = A(1 + r/n)^{nt}$ that shows the compounding period; and they work with functions written with the base e, $g(x) = Ae^{rt}$ , in many continuous growth contexts. In this section they do not build functions in any of these forms.

## M3.1.5 Modeling with exponential functions

**Build exponential functions to model real world contexts.**

Having seen the purpose of various different expressions for exponential functions, students now start to build functions in those forms in order model real-world contexts. Contexts may include Moore's law for computer processor speeds, population growth, and temperature change. Students should also consider whether an exponential or a linear model is appropriate in various contexts.

**Tasks**

F-LE In the Billions and Exponential Modeling
F-LE Boiling Water

## M3.1.6 Geometric sequences and series

- **Analyze situations that involve geometric sequences and series A-SSE.**
- **Derive the formula for the sum of a finite geometric series.**

Analyze situations that involve summing a exponential sequence (which is generally called a geometric series) These arise naturally in some saving and banking problems as well as in some interesting geometric contexts.

**Tasks**

A-SSE A Lifetime of Savings
A-SSE Course of Antibiotics
A-SSE Triangle Series
A-SSE Cantor Set

## M3.1.7 Summative assessment