M2.6 Right Triangle Trigonometry

• Using similarity, show that side ratios in right triangles are properties of the angles.
• Define the trigonometric ratios for acute angles.
• Explain and use the relationship between sine and cosine of complementary angles.
• Use trigonometric ratios to solve a variety of modeling problems.

After an open-ended task intended to motivate the need for trigonometric ratios (although the term is not used), students examine side ratios of similar right triangles. Students then build tables of side ratios for several similar triangles while noting that quotients of corresponding sides are the same. Using their knowledge of similar triangles they see that the ratios are a property of an angle rather than of a triangle. Only after laying this groundwork do students learn that, for right triangles, the ratios are called "sine," "cosine," and "tangent." Finally, students examine the relationship between sine and cosine and use this knowledge to solve a variety of problems.

In grades 6 and 7, students studied ratios. In those grades, a ratio is a pair of numbers. Two pairs are said to be "in the same ratio" if their quotients are equal. In previous geometry units, students have proved various theorems about triangles including "the sum of the measures of the interior angles of a triangle is 180°" and "the base angles of isosceles triangles are congruent." Students have established the AA criteria for similar triangles and have experience working with this theorem. At the end of unit G3, students used properties of similar triangles (proportionality of corresponding sides and congruence of corresponding angles) to solve problems. Traditionally, right angle trigonometry concerns quotients of side-lengths of triangles which are called ratios, thus identifying a pair of

numbers with its quotient. In this unit, if they haven't already done so, students make the transition to blurring distinctions between ratio as pair and ratio as quotient. After an open-ended task intended to motivate the need for trigonometric ratios (although the term is not used), they examine side ratios of similar right triangles. They build tables of side ratios for several similar triangles, noting that quotients of corresponding sides are the same. Using their knowledge of similar triangles, they see that the ratios are the property of an angle rather than of a triangle. In section 3, students learn that, for right triangles, the ratios are called "sine," "cosine," and "tangent." Students then examine the relationship between sine and cosine, and use their knowledge to solve problems. Later, students see the sine and cosine of an acute angle as the coordinates of a point on the unit circle formed by the terminal ray of the angle. They understand the radian measure of an angle as the arc subtended by the angle.F-TF.A.1 Using the unit circle, they extend the domains of the sine and cosine functions to all real numbers.F-TF.A.2 They make sense of the oscillations of sine and cosine, as well as the asymptotic behavior of tangent.

M2.6.0 Pre-unit diagnostic assessment

Assess students' ability to

• use the Pythagorean Theorem to find unknown lengths in right triangles;
• use properties of similar triangles to solve for an unknown side in similar right triangles.

After an open-ended task intended to motivate the need for trigonometric ratios (although the term is not used), students examine side ratios of similar right triangles. Students then build tables of side ratios for several similar triangles while noting that quotients of corresponding sides are the same. Using their knowledge of similar triangles they see that the ratios are a property of an angle rather than of a triangle. Only after laying this groundwork do students learn that, for right triangles, the ratios are called "sine," "cosine," and "tangent." Finally, students examine the relationship between sine and cosine and use this knowledge to solve a variety of problems.

M2.6.1 Why are trigonometric ratios useful?

Use side ratios of right triangles to solve a problem.

After an open-ended task intended to motivate the need for trigonometric ratios (although the term is not used), students examine side ratios of similar right triangles. Students then build tables of side ratios for several similar triangles while noting that quotients of corresponding sides are the same. Using their knowledge of similar triangles they see that the ratios are a property of an angle rather than of a triangle. Only after laying this groundwork do students learn that, for right triangles, the ratios are called "sine," "cosine," and "tangent." Finally, students examine the relationship between sine and cosine and use this knowledge to solve a variety of problems.

M2.6.2 Side ratios of similar triangles

Understand that, due to properties of similar triangles, side ratios in right triangles are properties of the angles in the triangle.

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M2.6.3 Definitions of sine and cosine

Formalize their understanding of the ratio table by defining the trigonometric ratios.

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M2.6.4 Relationship of sine and cosine

Explain and use the relationship between sine and cosine in complementary angles.

After an open-ended task intended to motivate the need for trigonometric ratios (although the term is not used), students examine side ratios of similar right triangles. Students then build tables of side ratios for several similar triangles while noting that quotients of corresponding sides are the same. Using their knowledge of similar triangles they see that the ratios are a property of an angle rather than of a triangle. Only after laying this groundwork do students learn that, for right triangles, the ratios are called "sine," "cosine," and "tangent." Finally, students examine the relationship between sine and cosine and use this knowledge to solve a variety of problems.

M2.6.5 Solve problems using sine and cosine

Use sine, cosine, and tangent to solve right triangles in applied problems.

After an open-ended task intended to motivate the need for trigonometric ratios (although the term is not used), students examine side ratios of similar right triangles. Students then build tables of side ratios for several similar triangles while noting that quotients of corresponding sides are the same. Using their knowledge of similar triangles they see that the ratios are

a property of an angle rather than of a triangle. Only after laying this groundwork do students learn that, for right triangles, the ratios are called "sine," "cosine," and "tangent." Finally, students examine the relationship between sine and cosine and use this knowledge to solve a variety of problems.

M2.6.6 Summative assessment

Students demonstrate their ability to

• use the relationship between sine and cosine to solve problems;
• use sine, cosine, and tangent to solve problems in right triangles (including indirect measurement).

After an open-ended task intended to motivate the need for trigonometric ratios (although the term is not used), students examine side ratios of similar right triangles. Students then build tables of side ratios for several similar triangles while noting that quotients of corresponding sides are the same. Using their knowledge of similar triangles they see that the ratios are a property of an angle rather than of a triangle. Only after laying this groundwork do students learn that, for right triangles, the ratios are called "sine," "cosine," and "tangent." Finally, students examine the relationship between sine and cosine and use this knowledge to solve a variety of problems.