M2.3 Quadratic Equations

- **Connect solving quadratic equations to finding zeros of quadratic functions.**
- **Explore forms of quadratic equations that can be solved by seeing structure.**
- **Understand and be able to use the method of factoring to solve factorable quadratic equations.**
- **Understand and be able to use the method of completing the square to solve quadratic equations, and derive the quadratic formula.**
- **Construct and solve quadratic equations by the most strategic method to solve problems in various contexts.**
- **Express a quadratic function in the appropriate form for a given purpose, including vertex form.**
- **Solve problems using systems consisting of a linear and a quadratic equation in two variables.**
- **Derive the equation of a parabola given the focus and a directrix parallel to one of the axes.**

Students have just completed a study of quadratic functions where they explored tables, graphs, and expressions for quadratic functions in situations that can be modeled by them. The various forms of a quadratic expression were necessitated and explored.

In this unit, students see quadratic equations arise naturally out of modeling problems involving quadratic functions. Any time you want to know the input to a quadratic function that produces a specified output, you have to solve a quadratic equation. Students understand solving quadratic equations as a process of reasoning and use the properties of operations to form equivalent quadratic expressions and equations. They convert between standard, vertex, and factored form by factoring, completing the square, and distributing. This unit does not treat factoring as a systematic method, but rather as an opportunistic method to be used when a factorization is readily available. The method of completing the square is a systematic method that works in all cases,

and leads to the quadratic formula. Students should be able to solve quadratic equations by many different methods, making strategic choices of the best method for the situation at hand.

Students then adapt the method of completing the square to putting a quadratic function in vertex form. The unit ends with a section on deriving the equation for a parabola from its geometric definition.

After this unit students will start to encounter situations where the roots are not real numbers, which will prepare them for a full investigation of complex numbers in the next unit.

M2.3.0 Pre-unit diagnostic assessment

Diagnose students' ability to
 • **recognize a solution to a quadratic equation;**
 • **perform operations on rational numbers;**
 • **work with irrational numbers, primarily by approximating on the number line.**

M2.3.1 Motivate solving quadratic equations

 • **Generate an intellectual need to solve a quadratic equation.**
 • **Understand the graphical method for solving equations approximately.**

In this section students develop an intellectual need for a general method for solving quadratic equations, through an encounter with a problem that is not obviously modeled by a quadratic equation and where the solution isn't obvious through reasoning or inspection. They solve the problem approximately using graphing technology. Some students may have experience with solving quadratic equations and offer solutions. Solving the equation exactly can be delayed until Section 3, where students can solve it by completing the square.

M2.3.2 What do we already know about solving quadratic equations?

 • **Understand the Zero Product Property and use it to justify steps to solve a factorable quadratic equation.**
 • **Explore forms of quadratic equations that can be solved by seeing structure.**
 • **Connect solving quadratic equations to finding zeros of quadratic functions.**

In Unit A3 students saw that the factored form of a quadratic function makes it easy to see the zeros of the function. Here they see that fact in a different light, learning that finding the zeros of a quadratic function is the same as solving the quadratic equation obtained by setting the function equal to zero. Students use factoring as an opportunistic method to solve quadratic equations, taking advantage of situations where a factorization is readily available from seeing the structure of a quadratic expression (e.g. a difference of squares) MP.7. Then they see how the vertex form of a quadratic function can also be useful in solving quadratic equations by taking square roots, as preparation for the general method of completing the square in the next section. Note: The fact that a number has at most two square roots is connected to the factored form. For example, the fact that $x^2 = 4$ has only the two solutions 2 and -2 follows from the fact that the equation $x^2 - 4 = 0$ can be converted to factored form $(x + 2)(x - 2) = 0.$

Tasks

A-REI Zero Product Property 1
A-REI, A-APR Solving a Simple Cubic Equation
Quadratic Sequence 1
Quadratic Sequence 2

M2.3.3 Solve quadratic equations by completing the square

Understand and be able to use the method of completing the square to solve quadratic equations.

This section continues to develop the method of completing the square with a carefully sequenced set of activities. Initially students look at

equations in which an expression of the form $x^2 + 2ax$ is clearly visible; then they consider equations in general form. By the end of the section students arrive at completing the square as a general method for solving quadratic equations, both executing the procedure and understanding how it works.

Tasks

Quadratic Sequence 3

A-REI Visualizing Completing the Square

M2.3.4 Put quadratic functions in vertex form.

Put quadratic functions in vertex form.

In this section, students adapt the method of completing the square from solving quadratic equations to putting into vertex form the expressions that define quadratic functions. This connects with the work in Unit A3 Section 7, where students saw the geometric meaning of vertex form but did not necessarily learn how to put a quadratic function in that form.

M2.3.5 Deriving the quadratic formula

Use completing the square to derive the quadratic formula.

Now that students have practice with completing the square, they have all of the tools they need to look at solving quadratic equations more abstractly and derive the quadratic formula. For some students, the algebraic manipulation in this section will seem overwhelming, but for others this section can bring home the connectedness between all they have learned about solving quadratic equations thus far. The main purpose of this section is to derive and build a conceptual understanding of the quadratic formula before using it to solve equations in the next section.

M2.3.6 Practice with all methods, picking the best one to use, and drawing connections between them.

• **Construct and solve quadratic equations by the most strategic method in**

various contexts.
 • Express a quadratic function in the appropriate form for a given purpose.

Now that students have a full toolbox for solving quadratic equations and expressing quadratic functions in different forms, their task becomes selecting which one to use in varying situations. In this section students see a variety of mathematical and real world problems, with the most strategic solution method varying from one problem to the next.
Tasks
A-REI Braking Distance
F-IF, A-REI Springboard Dive
F-IF Throwing Baseballs
A-REI Two Squares are Equal

M2.3.7 Connect algebra to geometry

 • Solve problems using systems of a linear and a quadratic equation in two variables.
 • Derive equation of a parabola given the focus and a directrix parallel to one of the axes.

In this section students connect equations in two variables to the geometry of the curves they define. They study systems of linear and quadratic equations. In Grade 8 they used similar triangles to explain why a line has constant slope, and thus derived the equation y=mx+b for a non-vertical line. In this unit they extend this work to parabolas, explaining why the geometric definition of a parabola leads to a quadratic equation in two variables.
Tasks
A-REI A Linear and Quadratic System
G-GPE Defining Parabolas Geometrically

M2.3.8 Culminating Activity

Apply the tools learned in this unit to a real-world situation.

In this section students have opportunities to strengthen and deepen their understanding by applying their new learning to a novel context.

M2.3.9 Summative Assessment

• **Assess students' ability to** • **reason with algebraic properties to solve quadratic equations;**
• **select the best method to solve a quadratic equation (factor, complete the square, quadratic formula);**
• **relate solutions of quadratic equations to graphs and interpret solutions in terms of a context.**