M1.5 Exponential Functions 1

• Distinguish between the growth laws of linear and exponential functions and recognize when a situation can be modeled by a linear function versus an exponential function.
• Graph exponential functions and understand how changing by a constant factor over equal intervals affects the graph.
• Model situations of growth and decay with exponential functions expressed in various different forms given a graph, a description of the situation, or two input-output pairs (including reading these from a table)
• Understand that over time a quantity increasing exponentially will eventually exceed a quantity increasing linearly.
• Understand the form of different expressions for exponential functions in terms of change by a constant factor over equal intervals.

In Grade 9, students should be familiar with linear functions from Grade 8 and from the F1 unit. They have been formally introduced to functions and function notation and have explored the behaviors and traits of both linear and non-linear functions. Additionally, students have spent significant time graphing, interpreting graphs and have explored how to compare the graphs of two linear functions to each other.

In this unit, students are introduced to exponential functions. Students learn the fundamental growth law for exponential functions and compare it with the law for linear functions. They recognize exponential functions when presented with data, graphs and real-world contexts. They construct exponential functions and use them to model situations and solve problems. They distinguish between situations that should be modeled with a linear function vs. an exponential function. They know various forms ways of expressing an exponential function (e.g. ,). They know that an increasing exponential function eventually is greater than an increasing linear function.

In A3, students will be introduced to quadratic functions. Students will focus on the basic nature of quadratic functions, contexts in the real world that can be modeled by quadratic functions, and the different forms of the expression for a quadratic function and what those forms tell you about the behavior of the function and the shape of its graph. Students will also extend their understanding of exponential functions and how they relate to quadratic functions; understanding that an exponential growth function will eventually exceed both a linear and a quadratic function. In the A7 unit, students will focus on a more in-depth understanding of exponential and log functions.

M1.5.0 Pre-unit diagnostic assessment

Diagnose students' ability to

- **use the exponent laws to find equivalent expressions;**
- **solve problems involving percent increase/decrease;**
- **solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form;**
- **construct a linear function;**
- **describe a non-linear function;**
- **compare functions**

M1.5.1 The Key to Exponential Growth

Learn the difference between growth by a constant multiplicative factor and growth by a constant additive factor.

In this section, students are introduced the underlying growth law for an exponential function, namely that the output changes by a constant multiplicative factor for a constant additive change in the input variable. They gain a quantitative sense of the difference this makes through an application to population growth.

Tasks

F-LE, A-CED Paper Folding

M1.5.2 Introduction to exponential functions

- **Distinguish between the growth laws of linear and exponential functions.**
- **Construct simple exponential models.**
- **Create tables and graphs of exponential functions and understand their**

behavior in terms of the fundamental growth law.
• Understand the form of the expression $f(x) = ab^x$ for an exponential function in terms of the fundamental growth law.

In this section students construct and interpret exponential functions expressed in the form f(x)=ab^x to model various context. They work with contexts where the initial value a and the growth factor b are either given or are directly inferable from the context, or where they must interpret those values in terms of the context. They make tables and graphs of exponential functions and begin to acquire a quantitative sense of exponential growth both numerically and graphically.
Tasks
F-LE U.S. Population 1790-1860
F-IF Identifying Exponential Functions
F-IF Exponential Parameters
F-LE Basketball Rebounds
F-LE Equal Differences over Equal Intervals 1
F-LE Equal Factors over Equal Intervals

M1.5.3 Model with Exponential Functions

• Model situations of growth and decay with exponential functions expressed in various different forms given a graph, a description of the situation, or two input-output pairs (including reading these from a table).
• Recognize that exponential functions have a constant percent growth or decay rate per unit interval.
• Interpret the parameters of an exponential function in a context.

Now that they are familiar with the basic form $f(x) = ab^x$ of an exponential function, students start to work with exponential functions expressed in other ways. They learn the relationship between the growth (or decay) factor and the growth (or decay) rate; if r is the growth rate then $1 + r$ is the growth factor. They model more complex situations where they must derive the growth factor in various ways given data about the context.
Tasks
F-LE Predicting the Past
F-LE Moore's Law and Computers

F-LE DDT-cay
F-LE All Your Base Are Belong to Us

M1.5.4 Compare exponential and linear functions

Understand that over time a quantity increasing exponentially will eventually exceed a quantity increasing linearly.

In this section, students will compare linear and exponential functions. Students are familiar with linear functions and linear growth so students will use this base understanding to develop the notion that increasing exponential functions will eventually exceed increasing linear functions.
Tasks
F-LE Linear or exponential?
F-LE, A-REI Population and Food Supply
F-LE Exponential growth versus linear growth I
F-LE Exponential growth versus linear growth II

M1.5.5 Bringing it Together

- **Interpret graphs and expressions for exponential functions.**
- **Model depreciation with linear and exponential functions.**
- **Compare linear and exponential models.**

In this section, students bring together much of what they have learned in this unit. They use their ability to write linear and exponential functions given two input-output pairs, they compare different models, both linear and exponential, and draw conclusions from them.

M1.5.6 Summative assessment

Assess students' ability to
- **construct and compare linear and exponential functions given data;**
- **recognize a situation in which a quantity grows by a constant percent rate per unit interval relative to another;**
- **given an exponential model in the form $f(t) = ab^t$, interpret the constants**

a **and** b **in terms of the context;**

• explain in words the similarities and differences between linear and exponential models;

• recognize situations that can be modeled with linear functions and with exponential functions, and solve problems.