M1.4 Functions

• Interpret key features of graphs in terms of the quantities represented. • Sketch graphs showing key features of the graph by hand and using technology. • Understand that a function from one set (the domain) to another set (the range) assigns to each element of the domain exactly one element of the range. • Use function notation. • Interpret statements that use function notation in various contexts. • Work with graphs of piecewise-defined functions, including step functions. • Relate the domain of a function to its graph. • Relate the domain of a function to the quantitative relationship it describes. • Calculate and interpret the average rate of change of a function over a specified interval. • Estimate the average rate of change of a function from its graph. • Solve for x such that f(x) = c, when f is a linear function. • Write an expression for the inverse of a linear function.

In grade 8, students are first introduced to the notion of functions. They understand a function as a rule that assigns to each input exactly one output 8.F.A.1. The main focus is on • linear functions and their representations (equations, graphs, tables, or verbal descriptions); • understanding that the equation y = mx + b defines a linear function, whose graph is a line; • modeling a linear relationship by a function, determining rate of change and initial value and interpreting them in terms of a situation modeled by the function and in terms of its graph or a table of values; • describing qualitatively a functional relationship between two quantities by analyzing a graph, e.g., where it is increasing or decreasing, linear or nonlinear. In this unit, students begin to use formal notation for functions, writing equations such as $f(x) = 2x + 3$ to describe a function. Students develop the understanding that the input/output relationship is a correspondence between two sets, and use the terms domain and range to describe them. They develop fluency with function notation and its use in

describing qualitative features of the graph of a function by first interpreting, then writing expressions, equations, and inequalities such as $f(x+2)$, $f(a)=2$, $f(x)>2$, and $f(x)>g(x)$. Students expand their repertoires of functions, working with piecewise-defined functions, including step functions. Building on their experiences with rate of change and slope from grade 8, students examine the behavior of non-linear functions. They describe key aspects of their graphs, and calculate and interpret average rates of change over specified intervals. Students' work with domain and range provides a basis for understanding when a function has an inverse. They examine simple functions that do and do not have inverses and write expressions for inverses of linear functions, but do not draw general conclusions about when a function has an inverse. Functions are a unifying theme in high school mathematics. In statistics, functions play especially prominent supporting roles as lines of best fit for bivariate statistics (units S2 and S4) and the normal distribution (unit S4). In geometry, transformations are viewed as functions sending points in the plane to points in the plane (units G1, G2, and G3), and ratios of sides of right triangles lead to the trigonometric functions on acute angles (unit G4 and A9). In algebra, students study systems of linear equations and inequalities (unit A1), exponential functions and geometric sequences (units A2 and A6), quadratic functions (unit A3), and rational functions, polynomials, and logarithms (unit A7 and A8).

M1.4.0 Pre-unit diagnostic assessment

Assess students' ability to

- **identify functions and non-functions;**
- **identify linear and nonlinear functions;**
- **write an equation for the corresponding linear function when given two points on a line;**
- **find and interpret the rate of change when given a linear function that models a situation;**
- **interpret the graph of a function in terms of the situation it models.**

M1.4.1 Graphing and functions

- **Sketch graphs showing key features.**
- **Interpret key features of graphs in terms of the quantities represented.**

In this unit, students begin their formal study of functions. They are introduced to function notation and gain a more precise understanding of what it means to be a function. They learn how to interpret functions in a given context and how to analyze them using different representations. In this section, they begin by graphing a variety of different functions.

M1.4.2 Introducing function notation

Understand that a function assigns to each element in the domain exactly one element of the range.

In this section, students are introduced to function notation and begin to interpret statements that use it. They begin to build expertise in understanding how equations and inequalities that use function notation correspond to features of graphs of functions. In this section, the focus is on statements about one input and its output (i.e., one point on the graph). In section 7, they return to this correspondence, going in the opposite direction: from features of graphs to equations and inequalities about the functions represented.

Tasks

F-IF Interpreting the graph
F-IF Using Function Notation I
F-IF The Customers
F-IF Points on a graph

M1.4.3 Interpreting function notation in context

Interpret statements that use function notation in terms of the quantities represented.

In this section, students apply and extend their understanding of function notation in various contexts by interpreting statements that use function notation in terms of the quantities represented.

Tasks

F-IF Cell phones
F-IF Yam in the Oven

M1.4.4 Mid-unit assessment

Assess students' ability to
• use function notation to represent points on a graph and to describe features of a graph;
• understand function notation and how to interpret statements in function notation in terms of a context.

M1.4.5 Domain, range, piecewise-defined functions

• Use the notions of domain and range.
• Interpret a graph of a piecewise-defined function.
• Graph step functions.

Tasks
F-IF The restaurant
F-IF Finding the domain
F-IF The Parking Lot
F-IF Bank Account Balance

M1.4.6 Interpreting graphs in context

• Interpret key features of graphs in terms of the quantities represented.
• Sketch graphs showing key features.
• Relate the domain of a function to its graph.
• Relate the domain to the quantitative relationship it describes.

In this section, students read and interpret graphs of functions. These include graphs of functions that arise from data, including functions that arise from using trend lines to join data points. Students also have an opportunity to apply their understanding of domain and range. The tasks in this section could be approached as a jigsaw activity. In small groups, students become experts on one particular task, then work in new groups which include at least one expert for each task.
Tasks
F-IF Oakland Coliseum

F-IF Warming and Cooling
F-IF Influenza epidemic
F-IF How is the Weather?
F-IF Telling a Story With Graphs

M1.4.7 Average rate of change

• Calculate the average rate of change of a function over a specified interval;
•Interpret the average rate of change of a function over a specified interval.

In this section, students are introduced to the notion of average rate of change over an interval. They work with expressions for average rates of change, and compute and estimate average rates of change. In grade 8, students learned that the rate of change of a linear function is the slope of its graph and that the slope can be computed from the coordinates of any two distinct points on the line. For nonlinear functions, the story is more complicated because their rates of change vary depending on the interval chosen. As for linear functions, average rates of change over an interval can be computed from the coordinates of two points on their graphs, namely those corresponding to the endpoints of the interval.
Tasks
F-IF Temperature Change
F-IF The High School Gym
F-IF Mathemafish Population

M1.4.8 Inverse functions

• Solve for x such that $f(x) = c$, when f is a linear function.
• Write an expression for an inverse.

WHAT: This post describes two days of instruction. First, students approach the idea of an inverse operation in the context of ciphers. They then write rules by listing out operations for a given function and then writing a new expression which “undoes” those operations F-BF.B.4a. The next day,

students create tables and graphs for functions and their inverses. They look for commonalities as they consider pairs of functions and their inverses. Note: This lesson was created for an Algebra 2 class, so depending on students' prior experience with various function types, it may need to be modified to make it appropriate for a given classroom. WHY: This lesson highlights fundamental characteristics about functions and their inverses: that it can generally be thought of as a process of "undoing," and also that the inputs and outputs trade places. This lesson does not address the idea that a function only has an inverse function if it is one to one, but that idea is addressed in later activities in this section.

Tasks

F-IF Your Father

F-BF Temperatures in degrees Fahrenheit and Celsius

F-BF US Households

M1.4.9 Summative assessment

Assess students' ability to

• identify the domain and range of a given function;

• create a step function that represents pairs of given values;

• interpret statements that use function notation in a context;

• identify key features of graphs;

• identify referents of expressions that use function notation in a given graph;

• identify and relate the domain and range of a function in a given context;

• calculate the average rate of change of a function over a specified interval;

• interpret the average rate of change of a function in terms of the quantities represented.