A2.5 Trigonometric Functions

• Understand some real-world situations that demonstrate periodic behavior.
• Define coordinates on the unit circle as the sine and cosine of an angle (F-TF.A.2).
• Understand radian measure and convert between radians and degree (F-TF.A.1).
• Graph basic trigonometric functions using radians as the x-axis scale (F-IF.C.7e *).
• Understand the relationship between parameters in a trigonometric function and the graph (F-IF.C.7e*).
• Model with trigonometric functions, including fitting them to data (F-TF.B.5*).
• Prove the Pythagorean Identity $\sin^2\theta + \cos^2\theta = 1$ (F-TF.C.8).
• Use the unit circle to prove trigonometric identities and relate them to symmetries of the graphs of sine and cosine (F-TF.A.4(+)).
• Use the Pythagorean Identity to calculate trigonometric ratios (F-TF.C.8).

Coming into this unit, students will already be familiar with right triangle trigonometry. They will have used sine, cosine and tangent to solve the angles and sides of a right triangle. Students will also have already seen that similar triangles preserve angles, as well as side ratios. For instance, they will have seen that a 3-4-5 right triangle has the same angles as a 6-8-10 triangle. They may have seen the “special right triangles” in the context of similar triangles. Students will also be familiar with Pythagorean Theorem, and will have used it to solve sides of right triangles. They will be familiar with the graphs of more than one type of function, including linear, quadratic, and exponential. Depending on timing, they may also have seen graphs of rational and polynomial functions.

In this unit, students make a big transition to thinking of trigonometric ratios as functions rather than a relationship between angles and side ratios in a right triangle. There are two important steps in this transition. First, students re-envision trigonometric ratios on the unit circle, and then use the idea of an angle as a movement around a circle to extend the definition to angles of any measure. Second, they use the unit circle to understand a new way of measuring angles, radian measure, which uses distance around the circumference of the circle to measure an angle, rather than an arbitrary division of the circle into 360 degrees. From now on students will be thinking of sine and cosine both as functions with numerical inputs, as well as ratios related to angles.

Students learn the basic shape of the graph of a trigonometric function, and then examine graphs of functions with parameters controlling the period, amplitude, and phase shift. They study the effect of varying these parameters and fit trigonometric functions to data. Addressing some (+) standards, students explore further the consequences of the unit circle definition of sine and cosine. They make a connection to the Pythagorean theorem. Then, they see how the symmetries of the circle give rise to symmetries of the graphs of sine and cosine, and represent these symmetries as identities.

In later math courses, students may see more complex uses of trigonometry. For instance, they may go on to learn that the derivative of the sinθ is the cosθ, and use radians to prove some of the properties of trigonometric derivatives.

A2.5.0 Pre-unit diagnostic assessment

Assess students' ability to
•using basic right triangle trigonometry to solve sides and angles of a right triangle (G-SRT.C.8⋆);
•use Pythagorean Theorem to solve for the sides of a right triangle (G-SRT.C.8⋆);
•view the vertical distance between the x-axis and a given point as the value of the point's y-coordinate, and the horizontal distance between the y-axis and a given point as the value of the point's x-coordinate.

A2.5.1 Introduction to periodic behavior

See a basic real-world model of periodic behavior and make sense of what data or graph it might generate.

Students have had a lot of exposure to graphing linear, quadratic, and exponential functions. They have yet to see a function that behaves periodically and understand how it might connect to a specific context. The ferris wheel provides a familiar scenario for students to see how the height of the cart will go up and down continuously, and to connect this information to a possible graph of the height. Students can make a rough sketch after watching the demo, or can use the more specific tools available in the Desmos activity to attempt to get a more accurate graph.

A2.5.2 Extending trigonometric functions to the real numbers

- **See coordinates on the unit circle as the sine and cosine of an angle (F-TF.A.2).**
- **Understand radian measure and convert between radians and degrees.**
- **Graph basic trigonometric functions using radians as the x-axis scale (F-IF.C.7e*).**

Students have understood trigonometric ratios in terms of right triangles, using them to solve for various sides and angles. In this section they make the big transition to thinking of trigonometric ratios as functions. There are two important steps in this transition. First, students re-envision trigonometric ratios on the unit circle, and then use the idea of an angle as a movement around a circle to extend the definition to angles of any measure. Second, they use the unit circle to understand a new way of measuring angles, radian measure, which uses distance around the circumference of the circle to measure an angle, rather than an arbitrary division of the circle into 360 degrees. From now on students will be thinking of sine and cosine as functions with numerical inputs, in addition to than ratios related to angles.

Tasks

F-TF.1 What exactly is a radian?
F-TF Bicycle Wheel
F-TF Trig Functions and the Unit Circle

A2.5.3 Modeling periodic behavior

- **Understand the relationship between parameters in a trigonometric function and the shape of the graph (F-IF.C.7e*).**
- **Model with trigonometric functions, including fitting them to data (F-TF.B.5*).**

Students have learned the basic shape of the graph of a trigonometric function, and now begin examining graphs of functions with parameters controlling the period, amplitude, and phase shift. They study the effect of varying these parameters and fit trigonometric functions to data.

Tasks

F-TF, F-BF Exploring Sinusoidal Functions

A2.5.4 Identities and special values for trigonometric functions

• Prove the Pythagorean Identity $\sin^2\theta + \cos^2\theta = 1$ (F-TF.C.8).
• Use the unit circle to prove trigonometric identities and relate them to symmetries of the graphs of sine and cosine (F-TF.A.4(+)).
• Use the Pythagorean Identity to calculate trigonometric ratios (F-TF.C.8).

In this section, which includes some (+) standards, students explore further the consequences of the unit circle definition of sine and cosine. They make a connection between the Pythagorean theorem. Then, they see how the symmetries of the circle give rise to symmetries of the graphs of sine and cosine, and represent these symmetries as identities.

Tasks

F-TF Trigonometric Ratios and the Pythagorean Theorem
F-TF Properties of Trigonometric Functions
F-TF, G-CO, Trigonometric Identities and Rigid Motions
F-TF Special Triangles 1

A2.5.5 Summative Assessment

Assess students' ability to
• understand radians and angles on a unit circle (F-TF.A.1, F-TF.A.3(+));
• graph basic and transformed trigonometric functions (F-IF.C.7e*);
• compare trigonometric functions algebraically and graphically (F-IF.C.9);
• interpret and graph data within a given context (F-TF.B.5*).

Tasks

F-TF Foxes and Rabbits 2