A2.2 Exponential Functions 2

- **Create and analyze a simple exponential function arising from a real-world or mathematical context (F-LE.A.2★).**
- **Evaluate and interpret exponential functions at non-integer inputs (N-RN.A.1, F-LE.A.2★).**
- **Understand functions of the form $f(t) = P(1 + r/n)^{nt}$ and solve problems with different compounding intervals (A-SSE.A.1★, F-LE.A.2★).**
- **Understand informally how the base e is used in functions to model a quantity that compounds continuously (F-BF.A.1a★).**
- **Write exponential expressions in different forms (F-LE.A.2★, F-BF.A.1★).**
- **Explain what the parameters of an exponential function mean in different contexts (F-LE.B.5★).**
- **Use the properties of exponents to write expressions in equivalent forms (A-SSE.B.3a★).**
- **Build exponential functions to model real world contexts (F-LE.A.1★, F-LE.A.2★, F-BF.A.1★).**
- **Analyze situations that involve geometric sequences and series (A-SSE.A.1★).**
- **Derive the formula for the sum of a finite geometric series (A-SSE.B.4★).**

In previous units students have worked with geometric sequences and understand they change by a constant ratio over a constant interval. They are able to write both recursive and closed equations for them. Students understand the difference between a linear and exponential function, can recognize situations and tables described by each, and know that an exponential function will always overtake a linear function. They know that an exponential function grows increasingly rapidly in one direction, and approaches a value asymptotically in the other direction. They have solved exponential equations of the form $ab^x = c$ by graphing. They can construct an exponential function given a graph, description of a relationship, or two input output pairs with integer inputs (including in a table). Given an expression defining an exponential function, they can interpret its parameters in a context. They can also fit a simple exponential function to a scatterplot. Every exponential function until now has only involved integer inputs.

In this unit students broaden their view of exponential functions to include the entire real number line as a possible domain. They learn about functions with base e. An approach using compound interest that shows e arising as the natural base for a quantity being compounded continuously can serve as a way to develop understanding appropriate to this level. First, students must understand functions of the form $f(x) = P(1 + r/n)^{nt}$ which show a given compounding frequency, n.

Students examine some different forms of exponential functions and learn to interpret the parameters in terms of a context. They learn the concept of doubling time and see functions expressed in a form that shows the doubling time; they work algebraically with functions expressed in a form like $f(x) = A(1 + r/n)^{nt}$ that shows the compounding period; and they work with functions written with the base e, $g(x) = Ae^{rt}$, in many continuous growth contexts. Students build functions in those forms in order model real-world contexts. Contexts may include Moore's law for computer processor speeds, population growth, and temperature change. Students should also consider whether an exponential or a linear model is appropriate in various contexts.

Students analyze situations that involve summing an exponential sequence (which is generally called a geometric series) These arise naturally in some saving and banking problems as well as in some interesting geometric contexts. Students will build on previous work with geometric sequences to derive a formula for the sum of a geometric series.

A later unit introduces logarithms as the solutions to exponential equations. Ultimately, students are proficient at graphing, analyzing, solving, and modeling exponential situations. Students are comfortable with exponentials from a functions viewpoint. This lays the foundations for success in calculus.

A2.2.0 Pre-unit diagnostic assessment

Diagnose students' ability to
- **create basic exponential functions (F-LE.A.2★);**
- **create an exponential model for a simple context (F-BF.A.1★);**
- **apply exponent rules (N-RN.A.1).**

A2.2.1 Understanding exponential growth and decay

Create and analyze a simple exponential function arising from a real-world or mathematical context (F-IE.A.2*).

Students have been introduced to exponential functions in Exponential Functions 1. For some students this introduction could have occurred a year or two previously. Therefore this unit starts with an activity that re-introduces exponential growth and decay in an engaging real-world or mathematical context. The context can be modeled by an exponential function with domain contained in the integers, thus providing a review of previous experience as needed for students.

A2.2.2 Exponential functions on the real numbers

Evaluate and interpret exponential functions at non-integer inputs(N-RN.A.1, F-LE.A.2*).

In Exponential Functions 1 students considered exponential functions at integer inputs only. Now that they understand how to determine the value of b^x for any rational number x, they can approximate b^x to any degree of accuracy for any real number x. In this section they broaden their view of exponential functions to include the entire real number line as a possible domain. They evaluate functions and interpret their values at real inputs in terms of a context, in preparation for the more sophisticated work in the following sections.

Tasks
F-LE Allergy medication
F-LE Boom Town

A2.2.3 Changing compounding intervals, continuous compounding, and the base e

- **Understand functions of the form $f(t) = P(1 + r/n)^{nt}$ which use a given compounding frequency, n (A-SSE.A.1*).**
- **Solve problems given different compounding intervals (F-LE.A.2*).**
- **Understand (informally) how the base e arises in the context of compounding intervals as n becomes arbitrarily large (F-BF.A.1a*).**

The base e is very commonly used in scientific and other modeling applications. The fundamental reason that e is a useful base for an exponential function is beyond the scope of this course. However, an approach using compound interest that shows e arising as the natural base for a quantity being compounded continuously can serve as a way to develop understanding at this level. First, students must understand functions of the form $f(t) = P(1 + r/n)^{nt}$ which show a given compounding interval, n. This section examines that approach.

Tasks

A-SSE The Bank Account
F-BF Compounding with a 5% Interest Rate

A2.2.4 Interpreting exponential functions

- **Solve problems involving exponential functions in many different contexts (F-LE.A.2*).**
- **Write exponential expressions in different forms (F-BF.A.1*).**
- **Explain what the parameters of an exponential function mean in different contexts (F-LE.B.5*).**
- **Use the properties of exponents to write expressions in equivalent forms (A-SSE.B.3c*).**

In Exponential Functions 1, students worked with exponential functions in the form $f(t) = ab^t$ or $f(t) = a(1 + r)^t$ and interpreted the parameters a, b, and r in terms of a context. In this unit they see more complicated forms. In the previous section, students developed an understanding of different compounding intervals and continuous compounding using base e. The purpose of this section is to examine some of these different forms and learn to interpret the parameters in terms of a context. Students learn the concept of doubling time and see functions expressed in a form that shows the doubling time; they work algebraically with functions expressed in a form like $f(x) = A(1 + r/n)^{nt}$ that shows the compounding period; and they work with functions written with the base e, $g(x) = Ae^{rt}$, in many continuous growth contexts. In this section they do not build functions in any of these forms.

Tasks

F-LE Bacteria Populations
F-BF Lake Algae

F-LE Rising Gas Prices – Compounding and Inflation
A-SSE Forms of exponential expressions

A2.2.5 Modeling with exponential functions

Build exponential functions to model real world contexts (F-LE.A.1★, F-LE.A.2★, F-BF.A.1★).

Having seen the purpose of various different expressions for exponential functions, students now start to build functions in those forms in order model real-world contexts. Contexts may include Moore's law for computer processor speeds, population growth, and temperature change. Students should also consider whether an exponential or a linear model is appropriate in various contexts.

Tasks
F-LE In the Billions and Exponential Modeling
F-LE Boiling Water

A2.2.6 Geometric sequences and series

- **Analyze situations that involve geometric sequences and series (A-SSE.A.1★).**
- **Derive the formula for the sum of a finite geometric series (A-SSE.B.4★).**

Analyze situations that involve summing a exponential sequence (which is generally called a geometric series) These arise naturally in some saving and banking problems as well as in some interesting geometric contexts.

Tasks
A-SSE A Lifetime of Savings
A-SSE Course of Antibiotics
A-SSE Triangle Series
A-SSE Cantor Set

A2.2.7 Summative assessment

Assess students' ability to
• evaluate and interpret exponential functions at non-integer inputs (N-RN.A.1, F-LE.A.2★);
• construct exponential functions given a description or data from a table (F-LE.A.2★);
• manipulate exponential expressions (A-SSE.B.3★);
• use the formula for the sum of a finite geometric series to solve a problem (A-SSE.B.4★);
• create and analyze simple exponential functions arising from real-word data (F-LE.B.5★).