A2.1 Extending the Number System

- **Work with infinite decimal expansions of numbers on the number line.**
- **Reason about operations with rational and irrational numbers (N-RN.B.3).**
- **Extend properties of integer exponents to rational exponents and write expressions with rational exponents as radicals (N-NR.A.1, N-RN.A.2).**
- **Solve equations and real-world problems involving radicals and fractional exponents (A-REI.A.2).**
- **Note extraneous solutions and explain where they come from (A-REI.A.2).**
- **Discover a new type of number that is outside previously known number systems (N-CN.A.1).**
- **Perform operations with complex numbers (N-CN.A.2).**
- **Solve quadratic equations with complex solutions (N-CN.C.7).**

In Grade 8, students discovered there are numbers that are not rational. They approximated them using rational numbers to locate them approximately on the number line. They also worked with integer exponents and became familiar with the basic exponent rules. Finally, in their work with quadratics, students encountered situations where they came to a negative under the square root sign, which was interpreted until now as meaning the equation had no solutions.

The theme of this unit is extending number systems and operations on numbers, first from rational to real numbers and then from real to complex numbers. Along the way students extend the operation of raising a number to a power to situations where the exponent is not an integer. They define the meaning of a numerical expression with rational exponents in terms of radicals, by reasoning about how to extend the exponent rules. They use this new understanding to solve equations involving rational exponents and radicals. Finally they extend the real number system and the operations on real numbers to include complex numbers and use them to solve quadratic equations with no real solutions.

In future units on exponential functions, students will extend their knowledge of

rational exponents and apply them to even more complicated situations. Students pursuing careers in STEM fields might use imaginary and complex numbers in many of their future college courses.

A2.1.0 Pre-unit diagnostic assessment

Assess students' ability to
• recall, from eighth grade, the definition of rational numbers and well-known examples of irrational numbers (8.NS.A.2);
• mentally compute and work with exponents, including 0, negative exponents, and negative bases (8.EE.A.1);
• solve a quadratic equation by various methods (A-REI.B.4).

A2.1.1 From rational numbers to the real number line

• Work with infinite decimal expansions of numbers on the number line.
• Reason about operations with rational and irrational numbers (N-RN.B.3).

In this section, students take a deeper look at the number line. In elementary school they learned how to place fractions on the number line and in middle school they added negative numbers and learned about the existence of a few isolated irrational numbers, such as π and $\sqrt{2}$. Now they see infinite decimal expansions as a way of locating any number, rational or irrational, on the number line. They also expand their repertoire of irrational numbers by considering sums and products of rational with irrational numbers.

Tasks
8.NS Identifying Rational Numbers
N-RN, 8-NS Calculating the square root of 2
N-RN Rational or Irrational?

A2.1.2 Extend the properties of exponents to rational exponents

• Extend properties of integer exponents to rational exponents and write expressions with rational exponents as radicals (N-RN.A.1, N-RN.A.2).
• Solve real-world problems in which rational exponents arise (N-RN.A.1).

Students have encountered square roots and cube roots in Grades 6–8. Now that they have the real numbers at their disposal they can contemplate more complicated numerical expressions involving radicals and fractional exponents. In this section they learn the rules for manipulating such expressions. They first review familiar exponent rules and remind themselves how they rewrite exponential expressions, particularly the rule $(x^a)^b = x^{ab}$. They investigate the consequences of extending this rule to rational exponents and see how it implies that $x^{(a/b)} = \sqrt[b]{x^a}$. The section continues with a modeling task using Kepler's Law and then wraps up with a short reasoning task where students can work with rational exponents in decimal form.

Tasks

N-RN Extending the Definitions of Exponents, Variation 2
N-RN Evaluating Exponential Expressions
N-RN Kepler's Third Law of Motion
N-RN Checking a calculation of a decimal exponent

A2.1.3 Solve radical equations

• Solve radical equations and equations with fractional exponents (A-REI.A.2).
• Note extraneous solutions and describe where they came from(A-REI.A.2).

In this section, students will move from reasoning about expressions with rational exponents to solving equations involving such expressions. They reason through to the solution step by step. They explore extraneous solutions and explain why they occur.

Tasks

A-REI Who wins the Race?
A-REI Radical Equations

A2.1.4 Mid-unit formative assessment

Assess students' ability to
• identify extraneous solutions when solving radical equations (A-REI.A.2);
• rewrite expressions with rational exponents and radicals (N-RN.A.2);
• demonstrate understanding about the outcomes of operations on rational

and irrational numbers (N-RN.B.3).

A2.1.5 Beyond the number line: complex numbers

Discover a new type of number that is outside previously known number systems (N-CN.A.1).

In this section, a new type of number is necessitated and then defined collaboratively. By hinging their understanding on their previous knowledge of numbers and then posing a question that doesn't fit into that system, students hit a point of disequilibrium where they need to define imaginary numbers and then seek to understand how they behave, particularly taking note of patterns that emerge with them.

Tasks
N-CN Complex number patterns

A2.1.6 Operations on complex numbers

• Explore how the new number, i, behaves under certain operations (N-CN.A.2).
• Perform operations with complex numbers and draw conclusions about patterns that emerge (N-CN.A.2).

With a new type of number in their world, students can begin exploring the properties of these numbers and how to add, subtract, multiply and divide them using the properties of operations to guide the work. They explore patterns that emerge when i is raised to positive integer powers, and build on this work by investigating similar patterns with other complex numbers.

Tasks
N-CN, A-SSE Computations with Complex Numbers
N-CN Powers of a complex number

A2.1.7 Solve quadratic equations with complex roots

Apply knowledge of complex numbers to solve quadratic equations with complex solutions (N-CN.C.7).

In this final section of the unit, students connect their knowledge of complex numbers to their previous work with quadratics. They learn that quadratic equations always have complex solutions even when they have no real solutions, unifying the class of quadratic equations when the complex number system is considered.

Tasks

N-CN, A-REI Completing the square

A2.1.8 Summative assessment

Assess students' ability to

• solve radical equations and demonstrate awareness of the possibility of extraneous roots resulting from an algebraic solution (A-REI.A.2);

• describe and show examples of how rational and irrational numbers behave under certain operations (N-RN.B.3);

• rewrite expressions involving rational exponents and radicals (N-RN.A.2);

• demonstrate understanding of the meaning of *i* (N-CN.A.1);

• perform operations with *i* and with complex numbers in the form *a* + *bi* (N-CN.A.2);

• solve quadratic equations with real coefficients that have complex solutions (N-CN.C.7).