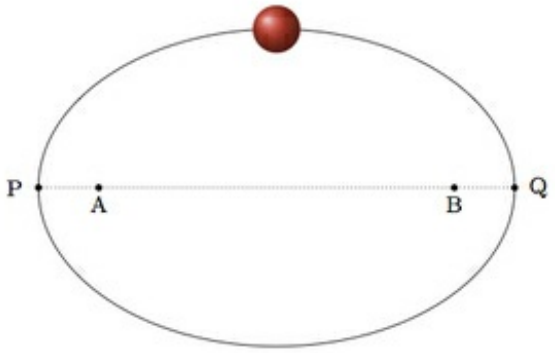
N-RN Kepler's Third Law of Motion

Task

Below is a picture of the (elliptical) orbit of a planet around the sun:



The sun is at point A, point P is where the planet is closest to the sun during its orbit, and point Q is where the planet is farthest from the sun during its orbit. Kepler made the following amazing discovery: if a is the average of the closest and farthest distances of the planet from the sun and t is the time it takes the planet to make one full orbit around the sun then the quotient $\frac{t^2}{a^3}$ does not depend on the planet. In what follows, we measure t in years and a in astronomical units where one astronomical unit is the average of the closest and farthest distances from the earth to the sun. These units are helpful since we have $t = 1$ and $a = 1$ for the earth, allowing us to find the quotient $\frac{t^2}{a^3} = 1$.

a. Find an equation for t in terms of a and an equation for a in terms of t.

b. The orbit of Mars takes about 1.88 years. What is its average distance from the sun?

c. The average distance of Neptune is 30.06 astronomical units. About how long does each orbit of Neptune take?

d. What is the farthest a planet could be from the sun during its orbit if each orbit takes 5 years?