7.NS.2d Decimal Expansions of Fractions

Task

Sarah learned that in order to change a fraction to a decimal, she can use the standard division algorithm and divide the numerator by the denominator. She noticed that for some fractions, like $\frac{1}{4}$ and $\frac{1}{100}$, the algorithm terminates at the hundredths place. For other fractions, like $\frac{1}{8}$, she needed to go to the thousandths place before the remainder disappears. For other fractions, like $\frac{1}{3}$ and $\frac{1}{6}$, the decimal does not terminate. Sarah wonders which fractions have terminating decimals and how she can tell how many decimal places they have.

a. Convert each of the following fractions to decimals to help Sarah look for patterns with her decimal conversions:

\[
\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{10}, \frac{1}{11}, \frac{1}{12}, \frac{1}{15}.
\]

b. Which fractions on the list have terminating decimals (decimals that eventually end in 0's)? What do the denominators have in common?

c. Which fractions on the list have repeating decimals? What do the denominators have in common?

d. Which fractions $\frac{p}{q}$ (in reduced form) do you think have terminating decimal representations? Which do you think have repeating decimal representations?