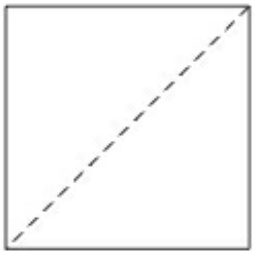
8.G, G-CO Origami Silver Rectangle

Task

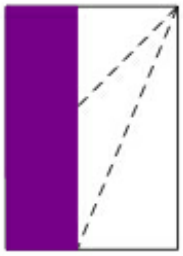
This task examines the mathematics behind an origami construction of a rectangle whose sides have the ratio $(\sqrt{2} : 1)$. Such a rectangle is called a *silver rectangle*.

Beginning with a square piece of paper, first fold and unfold it leaving the diagonal crease as shown here:

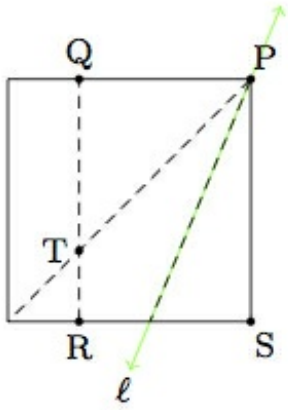


Next fold the bottom right corner up to the diagonal:

After unfolding then fold the left hand side of the rectangle over to the crease from the previous fold:



Here is a picture, after the last step has been unfolded, with all folds shown and some important points marked. In the picture T is the reflection of S about ℓ.



a. Suppose s is the side length of our square. Show that $|PT| = s$.

b. Show that $\triangle PQT$ is a 45-45-90 isosceles triangle.

c. Calculate $|PQ|$ and conclude that $PQRS$ is a silver rectangle.