N-CN Complex number patterns

Task

For this task, the letter i denotes the imaginary unit, that is, $i = \sqrt{-1}$.

a. For each integer k from 0 to 8, write i^k in the form $a + bi$.

b. Describe the pattern you observe, and algebraically prove your observation. In particular, simplify i^{195}.

c. Write each of the following expression in the form $a + bi$:

- $i^2 + i + 1$
- $i^3 + i^2 + i + 1$
- $i^4 + i^3 + i^2 + i + 1$
- $i^5 + i^4 + i^3 + i^2 + i + 1$
- $i^6 + i^5 + i^4 + i^3 + i^2 + i + 1$
- $i^7 + i^6 + i^5 + i^4 + i^3 + i^2 + i + 1$
- $i^8 + i^7 + i^6 + i^5 + i^4 + i^3 + i^2 + i + 1$

d. Describe the pattern you observe, and algebraically prove your observation. In particular, compute

$$i^{195} + i^{194} + \cdots + i^2 + i + 1.$$