8.EE Extending the Definitions of Exponents, Variation 1

Task

Marco and Seth are lab partners studying bacterial growth. They were surprised to find that the population of the bacteria doubled every hour.

a. The table shows that there were 2,000 bacteria at the beginning of the experiment. What was the size of population of bacteria after 1 hour? After 2, 3 and 4 hours? Enter this information into the table:

<table>
<thead>
<tr>
<th>Hours into study</th>
<th>Population (thousands)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. If you know the size of the population at a certain time, how do you find the population one hour later?

c. Marco said he thought that they could use the equation \( P = 2t + 2 \) to find the population at time \( t \). Seth said he thought that they could use the equation \( P = 2 \cdot 2^t \). Decide whether either of these equations produces the correct populations for \( t = 1, 2, 3, 4 \).

d. Assuming the population doubled every hour before the study began, what was the population of the bacteria 1 hour before the students started their study? What about 3 hours before?

e. If you know the size of the population at a certain time, how do you find the population one hour earlier?
f. What number would you use to represent the time 1 hour before the study started? 2 hours before? 3 hours before? Finish filling in the table if you haven't already.

g. Now use Seth's equation to find the population of the bacteria 1 hour before the study started. Use the equation to find the population of the bacteria 3 hours before. Do these values produce results consistent with the arithmetic you did earlier?

h. Use the context to explain why it makes sense that $2^{-n} = (\frac{1}{2})^n = \frac{1}{2^n}$. That is, describe why, based on the population growth, it makes sense to define $2$ raised to a negative integer exponent as repeated multiplication by $\frac{1}{2}$. 