6.EE Exponent Experimentation

Task

Decide whether each equation is true or false, and explain how you know.

a. \(2^4 = 2 \cdot 4\)

b. \(3 + 3 + 3 + 3 + 3 = 3^5\)

c. \(5^3 = 5 \cdot 5 \cdot 5\)

d. \(2^3 = 3^2\)

e. \(16^1 = 8^2\)

f. \((1 + 3)^2 = 1^2 + 3^2\)

g. \(2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = 6^3\)

IM Commentary

The purpose of this task is to give students experience working with exponential expressions and to promote making use of structure (MP7) to compare exponential expressions. To this end, encourage students to rewrite expressions in a different form rather than evaluate them to a single number. This may be best accomplished with a demonstration before students begin the task, like
Is $4^2 = 2^3$ true? Well, let’s see. I can rewrite each side like this:

$$4 \cdot 4 = 2 \cdot 2 \cdot 2$$

Then I can replace one of those $2 \cdot 2$'s with a $4$, like this:

$$4 \cdot 4 = 4 \cdot 2.$$ 

Now I can tell that this equation is not true.

For students who are accustomed to viewing the $=$ sign as a directive that means "perform an operation," tasks like these are essential to shifting their understanding of the meaning of the $=$ sign to one that supports work in algebra, namely, "The expressions on either side have the same value."

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Solution

a. $2^4 = 2 \cdot 4$ is false because it says $2 \cdot 2 \cdot 2 \cdot 2 = 2 \cdot 4$ or $16 = 8$.

b. $3 + 3 + 3 + 3 + 3 = 3^5$ is false because it says $3 + 3 + 3 + 3 + 3 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ or $15 = 243$.

c. $5^3 = 5 \cdot 5 \cdot 5$ is true because it says $5 \cdot 5 \cdot 5 = 5 \cdot 5 \cdot 5$ or $125 = 125$.

d. $2^3 = 3^2$ is false because it says $2 \cdot 2 \cdot 2 = 3 \cdot 3$ or $8 = 9$.

e. $16^1 = 8^2$ is false because it says $16 = 8 \cdot 8$ or $16 = 64$.

f. $(1 + 3)^2 = 1^2 + 3^2$ is false because it says $4^2 = 1 + 9$ or $16 = 10$.

g. $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = 6^3$ is true. We can use the meaning of exponents and the commutative and associative properties of multiplication to show this.
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6^3 = 6^3