5.NF To Multiply Or Not to Multiply, Variation 2

Alignments to Content Standards:  5.NF.A  5.NF.B.6

Task

Some of the problems below can be solved by multiplying $\frac{1}{8} \times \frac{2}{5}$, while others need a different operation. Select the ones that can be solved by multiplying these two numbers. For the remaining, tell what operation is appropriate. In all cases, solve the problem (if possible) and include appropriate units in the answer. In all cases, solve the problem (if possible) and include appropriate units in the answer.

a. Two-fifths of the students in Anya's fifth grade class are girls. One-eighth of the girls wear glasses. What fraction of Anya's class consists of girls who wear glasses?

b. A farm is in the shape of a rectangle $\frac{1}{8}$ of a mile long and $\frac{2}{5}$ of a mile wide. What is the area of the farm?

c. A pizza is cut into 8 slices. There is $\frac{2}{5}$ of the pizza left. If Jamie eats another slice, $\frac{1}{8}$ of the original whole pizza, what fraction of the original pizza is left over?

d. In Sam's fifth grade class, $\frac{1}{8}$ of the students are boys. Of those boys, $\frac{2}{5}$ have red hair. What fraction of the class is red-haired boys.

e. Alex was planting a garden. He planted $\frac{2}{5}$ of the garden with potatoes and $\frac{1}{8}$ of the garden with lettuce. What fraction of the garden is planted with potatoes or lettuce?

f. The track at school is $\frac{2}{5}$ of a mile long. If Jason has run $\frac{1}{8}$ of the way around the track, what fraction of a mile has he run?
IM Commentary

This task was written as part of a collaborative project between Illustrative Mathematics, the Smarter Balanced Digital Library, and the Teaching Channel. The rationale for including this task within the series of tasks is to support students in understanding when a problem calls for the use of multiplication and when other operations are called for in the problem. Differentiating between multiplication and division of fractions may not be as intuitive as multiplication and division of whole numbers. This task will provide students with opportunities to examine a variety of problems and make generalizations as they understand the nuances of the problems that call for multiplication.

Using this task once students have had opportunities to solve a variety of problems dealing with operations with fractions will provide useful insight into student understanding, providing teachers with information to guide next instructional decisions.

Things to look for and implications for instruction: Do students make connections between what they know about whole numbers and operating with those numbers to what they know about fractions? Do students attend to the whole that's being referred to within the problem? Do students rely on key words to know when to multiply?

a. Students with an error on this problem may not understand that in some instances, when working with fractions less than one, multiplication can be used to find a part of a part of an unknown whole. This idea may not map on to student understanding of multiplication of whole numbers such as multiplication involving equal groups, multiplicative comparison, area and arrays. Using a visual representation, such as a set or area representation may support students in seeing this relationship. If students are unfamiliar with these representations, working with whole numbers first is recommended. Students may have trouble identifying the whole before realizing that it is unknown also. A next step in this problem could be to have students think in terms of changing wholes. For example, using pattern blocks, with the hexagon as the whole, the trapezoid is half and the equilateral triangle is \( \frac{1}{6} \), however if the trapezoid is the whole, now the triangle is \( \frac{1}{3} \).

b. Students with an error on this problem, may not have a complete understanding of the concept of area. Providing students with opportunities to revisit the concept of area with whole numbers and then making connections to area problems with fractional parts may be a next step. For students needing a concrete way to see this, try
using post-it notes as unit squares: One can make a rectangle with side lengths of 3 and 5 and see that there are 15 unit squares (post-its) as expected, then try with one fractional side length first, cutting the post-its (e.g. \( \frac{1}{2} \) for one side length and 5 for the other to see that we can rearrange the area to get \( 2 \frac{1}{2} \) full squares which is \( 5 \times \frac{1}{2} = \frac{5}{2} \)) then move to a fraction times a fraction. Teachers might also consider units such as miles and how fractional parts of miles can equate to area. This can also be an opportunity to make connections between area and linear representations.

c. This problem requires subtracting fractional parts from the whole of 1 pizza. Students may overlook the wording “from the original pizza.” Supporting students to consider the units within the problem and acting out the problem may help students see the action of taking away part of the pizza within the problem as well as attend to the units being removed and the whole they are removed from. Students may also get bogged down drawing fractions of circles, especially 5ths. Pointing out that a useful representation doesn't necessarily match how something looks in reality may be helpful for students.

d. Like problem a above, students that struggle with this problem may not understand that a fraction can be a part of a part of an unknown whole. Providing students with experiences to represent the story using a set or area model may help them visualize what is happening in the story and see that the problem is asking them to find a part (boys with red hair) of a part (boys in the class) of an unknown whole (students in fifth grade class).

e. This problem may be solved using addition as students are asked to figure out how much of the garden is planted with potatoes and lettuce. Students may not interpret these as separate quantities but instead be confused about overlapping areas. Using visual representations of the garden and labeling the parts given in the problem may support students in understanding the context. Unpacking the problem to clearly understand what the problem is asking may be a next step for students to connect the problem to the addition operation.

f. This problem can be solved with multiplication and provides an opportunity to connect student understandings of representation to a linear model. Students that struggle with this problem may not see the unit as one mile.

**Solutions**

**Edit this solution**

**Solution: a**
a. Multiplication is appropriate and $\frac{2}{40}$ or $\frac{1}{20}$ of the girls in the class wear glasses.

Solution: b

b. Multiplication is appropriate and the farm has an area of $\frac{2}{40}$ square mile.

Solution: c

c. This problem cannot be solved by multiplication. Instead we subtract $\frac{1}{8}$ from the $\frac{2}{5}$ that was available to find that $\frac{2}{5} - \frac{1}{8} = \frac{16}{40} - \frac{5}{40} = \frac{11}{40}$, of the pizza is left. The related question involving multiplication would be: “There is $\frac{2}{5}$ of a pizza left. If Jamie ate $\frac{1}{8}$ of that part of the pizza, what fraction of the original whole pizza would he have eaten?”

Solution: d

d. Multiplication is appropriate and $\frac{2}{40}$ or $\frac{1}{20}$ of the boys have red hair.

Solution: e

e. This is an addition problem. $\frac{2}{5} + \frac{1}{8} = \frac{21}{40}$ of the garden was planted in potatoes or lettuce.

Solution: f

f. Multiplication is appropriate and Jason has run $\frac{2}{40}$ or $\frac{1}{20}$ of a mile.