**Task**

Jessica gets her favorite shade of purple paint by mixing 1/3 cup of blue paint with 1/2 cup of red paint. How many cups of blue and red paint does Jessica need to make 20 cups of her favorite purple paint?

**IM Commentary**

The goal of this task is to provide a context for students to develop their ratio and proportional reasoning skills. Many of the techniques developed in the sixth grade can be readily applied here: for example, solution 2 below uses a ratio table. Other techniques, such as double number lines or tape diagrams, can also be applied (see solution 6) although the fractions make these more challenging. If the fractions in the problem (1/3 and 1/2) were more complex (1/3 and 1/5 for example) then there is genuine motivation to examine more abstract techniques. The numbers chosen make this a good bridge problem, allowing students to practice the techniques learned in the sixth grade while also working with the more abstract seventh grade ideas.

The sixth grade version of this problem, [https://www.illustrativemathematics.org/tasks/2049](https://www.illustrativemathematics.org/tasks/2049), replaces the fractions 1/3 and 1/2 in the prompt with whole numbers 2 and 3. If students are successful applying double number lines, tape diagrams, and ratio tables with fractions then they have demonstrated mastery of the sixth grade techniques and are ready to move into the more abstract ratio and proportion methods exemplified by solutions 3 and 4.

If this task is being used to motivate proportional reasoning techniques, the teacher may wish to give students more complex fractions than 1/2 and 1/3. As the numbers...
become more complex, physical representations such as tape diagrams and double number lines become more difficult to manipulate. Methods such as scaling (solution 1), setting up a proportion (solution 3), or setting up an equation (solution 4) work for any fractions, provided students are fluent doing fraction arithmetic.

In addition to being used to bridge sixth and seventh grade ratio and proportional thinking, this task can also be used after students have experience with the seventh grade language and techniques. In this case, a large variety of responses, as shown, should be encouraged and expected and the teacher may wish to have students share their different approaches. The vast array of techniques available to solve this problem are indicative of the central role of ratio and proportion in the middle school curriculum. Building upon arithmetic with fractions, it prepares students for using expressions and equations, graphing lines, and understanding the meaning of functions.

This task was developed with the assistance of a group of teachers from Washington and Illinois in connection with an SBAC digital library project. In the lesson, the statement of the problem was "If Perfect Purple Paint is made by mixing 1/3 cups blue paint to 1/2 cup red paint, how much of each is needed for 20 cups?"

This task was written as part of a collaborative project between Illustrative Mathematics, the Smarter Balanced Digital Library, the Teaching Channel, and Desmos.

**Solutions**

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**Solution: 1 Arithmetic and Scaling**

One batch of purple paint contains 1/3 cup of blue paint and 1/2 cup of red paint. This will make a total of \(\frac{1}{2} + \frac{1}{3} = \frac{5}{6}\) of a cup of purple paint. In order to make 20 cups of purple paint, we need \(20 \div \frac{5}{6} = 24\) batches. Each batch has 1/3 of a cup of blue paint so 24 batches will contain \(24 \times \frac{1}{3} = 8\) cups of blue paint. Each batch has 1/2 cup of red paint so 24 batches will contain \(24 \times \frac{1}{2} = 12\) cups of red paint.

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**Solution: 2 Ratio table**

We can use a ratio table to find how much blue and red paint will be in 20 cups of
Jessica's perfect purple paint:

<table>
<thead>
<tr>
<th>Blue Paint (cups)</th>
<th>Red Paint (cups)</th>
<th>Purple Paint (cups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/3</td>
<td>1/2</td>
<td>5/6</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>20</td>
</tr>
</tbody>
</table>

For the second row, we take 6 batches of the purple paint mixture, in order to get a whole number of cups of purple paint. From here, doubling this mixture twice shows that there are 8 cups of blue paint and 12 cups of red paint in 20 cups of purple paint.

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**Solution: 3 Proportions**

Combining 1/3 cup of blue paint and 1/2 cup of red paint makes 1/3 + 1/2 = 5/6 cups of blue paint. To find how much blue paint is in 20 cups of purple paint we can use a proportion:

\[
\frac{\frac{1}{3}}{\frac{5}{6}} :: \frac{\text{?}}{20}.
\]

Note that \(\frac{1}{3} = \frac{2}{6}\) so \(\frac{1}{3} \div \frac{5}{6} = \frac{2}{6} \div \frac{5}{6} = \frac{2}{5}\). So \(\text{?}\) satisfies the equation

\[
\frac{\text{?}}{20} = \frac{2}{5}.
\]

We can solve this equation to find \(\text{?} = 8\). There are 8 cups of blue paint in 20 cups of Jessica's purple paint.

Since the rest of the paint in the 20 cups of purple paint is red, this means that there are 12 cups of red paint in the purple paint mixture.

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**Solution: 4 Equations**
One batch of purple paint is $1/3 + 1/2 = 5/6$ cups. The amount of blue paint is $1/3 = 2/6$ cups. This means that there is $5/2$ times as much purple paint as blue paint. If $p$ is the amount of purple paint and $b$ the amount of blue paint, this means

$$p = \frac{5}{2} b.$$ 

So if we have 20 cups of purple paint then to find out how much blue paint there is we can solve

$$20 = \frac{5}{2} b$$

to find that there are 8 cups of blue paint.

Similarly, if $r$ denotes the red paint in the mixture then

$$p = \frac{5}{3} r$$

and if $p = 20$ then we find $r = 12$ so there are 12 cups of red paint in 20 cups of Jessica’s purple paint.

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**Solution: 5 Using Percent**

There are $1/3 + 1/2 = 5/6$ cups of purple paint in a single batch of Jessica’s favorite purple. Since $1/3 = 2/6$ this means that 2 out of 5 equal parts, or 40%, of the purple paint comes from blue paint. Since 40% of 20 cups is 8 cups, 8 cups of blue paint are needed to make 20 cups of purple paint.

Similarly, the remaining 60% of the purple paint comes from the added red paint. Since 60% of 20 cups is 12 cups, 12 cups of red paint are needed to make 20 cups of purple paint.

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**Solution: 6 Tape diagrams**

We put together batches of the purple paint until we find whole number of cups of both red paint and blue paint:
Here we have taken 6 batches, each consisting of $\frac{1}{3}$ cup of blue paint and $\frac{1}{2}$ cup of red paint. Together, this makes 5 cups of purple paint. So if we combine four of these that makes 20 cups of perfect purple paint. Four groups of 2 cups of blue paint make 8 cups of blue paint and four groups of 3 cups of red paint make 12 cups. So 20 cups of perfect purple paint contains 8 cups of blue paint and 12 cups of red paint.