

S.IC.4 Margin of Error for Estimating a Population Mean

Alignments to Content Standards: S-IC.B.4

Task

Background:

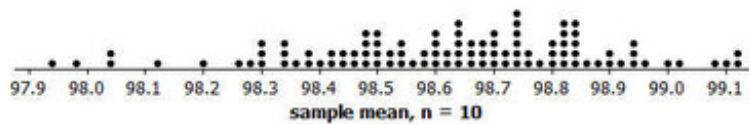
Researchers have questioned whether the traditional value of 98.6°F is correct for a typical body temperature for healthy adults. Suppose that you plan to estimate mean body temperature by recording the temperatures of the people in a random sample of 10 healthy adults and calculating the sample mean. How accurate can you expect that estimate to be? In this activity, you will develop a margin of error that will help you to answer this question.

Let's assume for now that body temperature for healthy adults follows a normal distribution with mean 98.6 degrees and standard deviation 0.7 degrees. Here are the body temperatures for one random sample of 10 healthy adults from this population:

97.73	98.76	98.27	99.95	98.47	98.49	98.97	98.68	99.27	99.25
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- What is the mean temperature for this sample?
- If you were to take a different random sample of size 10, would you expect to get the same value for the sample mean? Explain.

Below is a dot plot of the sample mean body temperature for 100 different random samples of size 10 from a population where the mean temperature is 98.6 degrees.



- c. How many of the samples had sample means that were greater than 98.5 degrees and less than 98.7 degrees?
- d. Based on the dot plot above, if you were to take a different random sample from the population, would you be surprised if you got a sample mean of 98.8 or greater? Explain why or why not.
- e. Which of the following statements is appropriate based on the dot plot of sample means above?

Statement 1: Most random samples of size 10 from the population would result in a sample mean that is within 0.1 degrees of the value of the population mean (98.6).

Statement 2: Most random samples of size 10 from the population would result in a sample mean that is within 0.3 degrees of the value of the population mean (98.6).

Statement 3: Most random samples of size 10 from the population would result in a sample mean that is within 0.5 degrees of the value of the population mean (98.6).

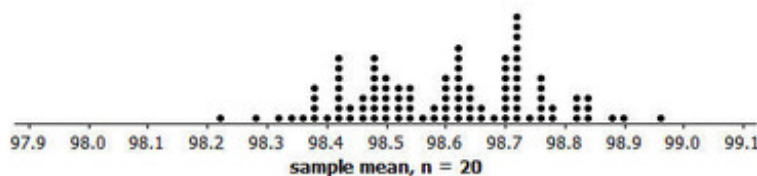
The **margin of error** associated with an estimate of a population mean can be interpreted as the maximum likely difference between the estimate and the actual

value of the population mean for a given sample size.

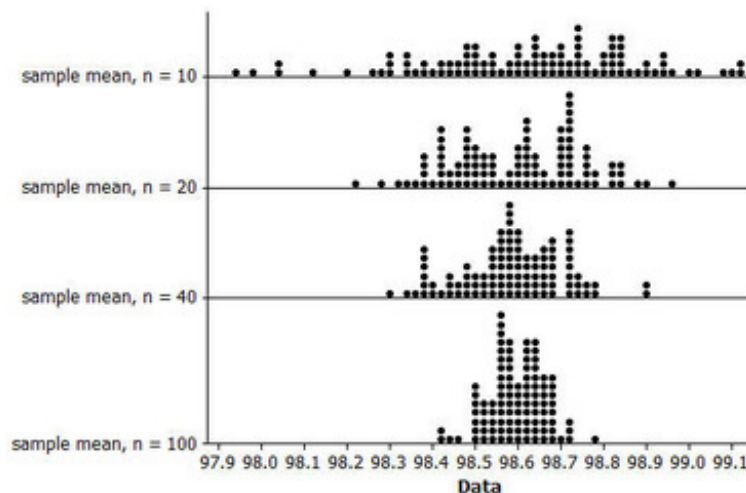
f. Explain why 0.45 degrees would be a reasonable estimate of the margin of error when using the sample mean from a random sample of size 10 to estimate the mean body temperature for the population described above.

g. If you were to use a random sample of size 20 to estimate the population mean, would you expect the estimate to be closer to or farther from the actual value of the population mean than if you had used a random sample of size 10? Would this mean that the margin of error would be less than or greater than the margin of error for a sample of size 10?

h. Below is a dot plot of the sample mean temperature for 100 different random samples of size 20 from a population with an actual mean temperature of 98.6 degrees. Explain how this dot plot supports your answer in part (g).



Below is a comparative dot plot that shows sample means for 100 random samples for each of the sample sizes 10, 20, 40, and 100.



i. What is a reasonable estimate of the margin of error for samples of size 20? For samples of size 40? For samples of size 100?

j. How is the margin of error related to sample size?

In practice, we don't take many random samples from a population and we don't know the actual value of the population mean, so we need a way to estimate the margin of error from a single sample.

An estimate of the margin of error based on a single random sample can be obtained by evaluating the following expression

$$\text{estimated margin of error} = 2 \frac{s}{\sqrt{n}}$$

where s is the sample standard deviation and n is the sample size.

k. Using the sample at the beginning of this activity, what is the estimated margin of error?

I. Suppose that a random sample of 50 healthy adults resulted in a sample mean body temperature of $\bar{x} = 98.2$ degrees and a sample standard deviation of $s = 0.65$ degrees. Would you consider this evidence that the actual mean temperature for healthy adults is in fact less than 98.6 degrees? (Hint: what is the estimated margin of error?)

IM Commentary

The purpose of this task is to illustrate the development of margin of error when estimating a population mean (S.IC.4). The results from several simulations are used to develop margin of error and then a way of estimating the margin of error from a single sample is introduced. This is a challenging task, but it is well aligned with standard S.IC.4, which is one of the more complex statistics standards.

[Edit this solution](#)

Solution

a. 98.78 degrees

b. No. The value of the sample mean will vary from sample to sample because different samples will include different individuals from the population.

c. 25

d. No. Many of the 100 random samples resulted in a sample mean that was 98.8 or

greater. This would not be surprising.

e. Statement 3 (although some students may say Statement 2, depending on how they interpret “most”).

f. Only 9 of the 100 samples had sample means that were farther from 98.6 degrees than 0.45 degrees. This means that 91 of the 100 random samples resulted in a sample mean that was closer to the actual value of the population mean than 0.45 degrees. It would be unlikely to get a random sample of size 10 that had a sample mean farther away from the population mean than 0.45 degrees.

g. Would expect it to be closer because the sample size of 20 is larger than the sample size 10. This would mean that we expect the margin of error to be smaller.

h. Most of the samples of size 20 were within about 0.25 degrees of the actual value of the population mean. This is smaller than 0.45 degrees.

i. Answers will vary, but one set of reasonable answers is 0.45, 0.25, 0.20, and 0.15 degrees.

j. The greater the sample size, the smaller the value of the margin of error.

k. The sample standard deviation is 0.62 degrees. The estimated margin of error is then

$$\text{estimated margin of error} = 2 \frac{s}{\sqrt{n}} = 2 \frac{(0.62)}{\sqrt{10}} = 0.392 \text{ degrees}$$

l. For a sample of size 50,

$$\text{estimated margin of error} = 2 \frac{s}{\sqrt{n}} = 2 \frac{(0.65)}{\sqrt{50}} = 0.184 \text{ degrees}$$

This means that it would be unlikely to get a sample mean that is farther from the actual population mean than 0.184 degrees. The sample mean of 98.2 degrees is 0.4 degrees away from 98.6 degrees, so it seems unlikely that the actual population mean could have been 98.6 degrees.



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