3.OA Valid Equalities? (Part 2)

Alignments to Content Standards: 3.OA.B.5

Task

Decide if the equations are true or false. Explain your answer.

a. $4 \times 5 = 20$

b. $34 = 7 \times 5$

c. $3 \times 6 = 9 \times 2$

d. $5 \times 8 = 10 \times 4$

e. $6 \times 9 = 5 \times 10$

f. $2 \times (3 \times 4) = 8 \times 3$

g. $8 \times 6 = 7 \times 6 + 6$

h. $4 \times (10 + 2) = 40 + 2$

IM Commentary

This task is a follow-up task to a first grade task:
http://www.illustrativemathematics.org/illustrations/466.

On the surface, both tasks can be completed with sound procedural fluency in addition and multiplication. However, these tasks present the opportunity to delve much more deeply into equivalence and strategic use of mathematical properties. These tasks add clarity to the often misunderstood or neglected concept of equivalence. Students often
understand the equal sign as the precursor to writing the answer. Class discussion should be carefully guided to ensure that students come to the understanding that the equal sign indicates equivalence between two expressions. Though these tasks can be completed by evaluating each expression on either side of the equal sign, they present deliberate next levels of reasoning that invite students to look for different approaches.

Anyone facilitating a conversation about this task should constantly ask, "Is there another way to know whether this equation is true?" Consider $5 \times 8 = 10 \times 4$. Students will likely know these facts relatively quickly and come to the conclusion that both sides are equal to 40, thus this equation is true. When pressed to see other options, students may reason that the 8 can be broken down into $4 \times 2$. The equation becomes $5 \times (2 \times 4) = 10 \times 4$. Through the associative property, this becomes $(5 \times 2) \times 4 = 10 \times 4$. We can see that these expressions are equivalent because we know that $5 \times 2$ has the same value as 10. The same opportunity presents itself in part f. Part g presents an opportunity for students to think critically about the meaning of multiplication.

Third graders interpret multiplication as equal sized groups. Students might reason that $8 \times 6$ means 8 groups of 6. Thus $7 \times 6 + 6$ would mean 7 groups of 6 with another group of 6. Students might recognize that extra 6 as the "8th group of 6," thereby making the two expressions equivalent.

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**Solution**

Decide if the equations are true or false. Explain your answer.

a. $4 \times 5 = 20$ This is true. Both sides have a value of 20.

b. $34 = 7 \times 5$ This is not true. The value of $7 \times 5$ is 35. Students may also recognize that all multiples of 5 end in 0 or 5 and conclude that this equation is not true without evaluating $7 \times 5$.

c. $3 \times 6 = 9 \times 2$ This is true. Both sides have a value of 18.

d. $5 \times 8 = 10 \times 4$ This is true. Both sides have a value of 40. Additional information from the Commentary: Students will likely know these facts relatively quickly and come to the conclusion that both sides are equal to 40, thus this equation is true. When pressed to see other options, students may reason that the 8 can be broken down down into $4 \times 2$. 

The equation becomes $5 \times (2 \times 4) = 10 \times 4$. Through the associative property, this becomes $(5 \times 2) \times 4 = 10 \times 4$. We can see that these expressions are equivalent because we know that $5 \times 2$ has the same value as 10.

e. $6 \times 9 = 5 \times 10$ This is not true. The value of $6 \times 9$ is 54 and the value of $5 \times 10$ is 50. This is included to see if students mistakenly try to use compensation strategies from addition.

f. $2 \times (3 \times 4) = 8 \times 3$ This is true. Both sides have a value of 24. See the commentary for further suggestions on what ideas to elicit from students.

g. $8 \times 6 = 7 \times 6 + 6$ This is true. Both sides have a value of 48. Additional information from the Commentary: third graders understand multiplication in terms of equal groups. Students might reason that $8 \times 6$ means 8 groups of 6. Thus $7 \times 6 + 6$ would mean 7 groups of 6 with another group of 6. Students might recognize that extra 6 as the "8th group of 6" thereby making the two expressions equivalent.

h. $4 \times (10 + 2) = 40 + 2$ This is not true. $4 \times (10 + 2)$ has a value of 48, and $40 + 2$ has a value of 42. This equation presents an opportunity for students to reason using the distributive property.