7.NS Why is a Negative Times a Negative Always Positive?

Alignments to Content Standards: 7.NS.A.2.a  7.NS.A.2

Task

Some people define $3 \times 5$ as $5 + 5 + 5$, which has a value of 15.

a. If we use the same definition for multiplication, what should the value of $3 \times (-5)$ be?

b. Here is an example of the distributive property:

$$3 \times (5 + 4) = 3 \times 5 + 3 \times 4$$

If the distributive property works for both positive and negative numbers, what expression would be equivalent to $3 \times (5 + (-5))$?

If we use the fact that $5 + (-5) = 0$ and $3 \times 5 = 15$, what should the value of $3 \times (-5)$ be?

c. We can multiply positive numbers in any order:

$$3 \times 5 = 5 \times 3$$

Use what you know from parts (a) and (b). If we can multiply signed numbers in any order, what should the value of $(-5) \times 3$ be?

If the distributive property works for both positive and negative numbers, what expression would be equivalent to $(-5) \times (3 + (-3))$?
d. Use what you know from parts (a), (b), and (c). What should the value of \((-5) \times (-3)\) be?

**IM Commentary**

The purpose of this task is for students to understand the reason it makes sense for the product of two negative numbers to be positive. The idea is that if the properties of operations with which we are familiar when we do arithmetic with positive numbers are universal, then we have to define multiplication on signed numbers the way we do. The task only works through a single example, but the argument would work for any two negative numbers.

Another popular way to explain the rules for multiplying signed numbers involves looking at patterns in multiples of, say, 5.

<table>
<thead>
<tr>
<th>5 \times 5</th>
<th>4 \times 5</th>
<th>4 \times 5</th>
<th>2 \times 5</th>
<th>1 \times 5</th>
<th>0 \times 5</th>
<th>(-1) \times 5</th>
<th>(-2) \times 5</th>
<th>(-3) \times 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>20</td>
<td>15</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If you note that the multiples are decreasing by 5 each time, then the natural way to continue the pattern is to fill in -5, -10, and -15 into the three empty cells in the table. Whether we are explicit about this or not, this argument also relies on the distributive property. For example,

\[
5 \times 5 = (4 + 1) \times 5
= 4 \times 5 + 1 \times 5
= 4 \times 5 + 5
\]

so

\[
5 \times 5 - 5 = 4 \times 5
\]

Similarly,

\[
0 \times 5 = (-1 + 1) \times 5
=(-1) \times 5 + 1 \times 5
=(-1) \times 5 + 5
\]
This task assumes students know how to add and subtract signed numbers, but isn't very interesting if students already know the rules for multiplying signed numbers since they will likely think to answer the questions by citing the rules rather than thinking about the reason for those rules (which is what the task is trying to get at). Even though the task is heavily scaffolded, students might have a hard time figuring out what they are supposed to do. Depending on students' tolerance for reading and deciphering problems of this nature, it might be advisable for the teacher to go through an example with different numbers first so students understand what is expected. A slightly different, more visual approach to these ideas can be found here: https://www.illustrativemathematics.org/tasks/1986. Both tasks focus on products of negative numbers and the distributive property; this task focuses on the arithmetic of the distributive property while the other emphasizes geometry.

### Solution

a. The value of $3 \times (-5)$ should be $(-5) + (-5) + (-5) = -15$.

b. Applying the distributive property gives

$$3 \times (5 + (-5)) = 3 \times 5 + 3 \times (-5).$$

Since $5 + (-5) = 0$ and $3 \times 5 = 15$,

$$3 \times 0 = 3 \times (5 + (-5))$$

$$= 3 \times 5 + 3 \times (-5)$$

$$= 15 + 3 \times (-5)$$

Since $3 \times 0 = 0$, these equations show that $0 = 15 + 3 \times (-5)$. From here, we see that $3 \times (-5) = -15$.

c. Extending the commutative property of multiplication to negative numbers, the product $(-5) \times 3$ should have the same value as $3 \times (-5)$ so it should be -15. Furthermore, the distributive property gives

$$(-5) \times (3 + (-3)) = (-5) \times 3 + (-5) \times (-3).$$
d. Using the equality from part (c),

\[ (-5) \times (3 + (-3)) = (-5) \times 3 + (-5) \times (-3), \]

we know the left side is 0 and we know \((-5) \times 3 = -15\). So

\[ 0 = -15 + (-5) \times (-3) \]

and \((-5) \times (-3) = 15\).