5.0A Using Operations and Parentheses

Alignments to Content Standards: 5.OA.A.1

Task

What numbers can you make with 1, 2, 3, and 4? Using the operations of addition, subtraction, and multiplication, we can make many different numbers. For example, we can write 13 as

\[ 13 = (3 \times 4) + 1. \]

You can use parentheses as many times as you like and each of the numbers 1, 2, 3, and 4 can be used at most once.

a. Find two different ways to make 9.

b. Find two different ways to make 7.

c. Find two different ways to make 11.

d. Can you make 26?

IM Commentary

The purpose of this task is to give students a chance to work creatively with three of the four fundamental arithmetic operations (addition, subtraction, and multiplication). It is well suited for helping students develop fluency with addition, subtraction, and multiplication of single digit numbers. If the teacher prefers, rather than asking to make 7 in two different ways, she could instead prompt the students to look for as many
different ways as possible to get 7. Students working on this task can improve their fluency with addition, subtraction, and multiplication while also gaining practice in carefully writing down the order of operations by using parentheses. The teacher may wish to do some examples to make sure that students understand what they are being asked to do: for example, going over many different ways to make 5 could be helpful: $5 = 4 + 1$, $5 = (2 \times 3) - 1$, $5 = (4 - 1) + 2$. If students try to put the numbers 1, 2, 3, and 4 together as digits in a larger number, for example writing $9 = 12 - 3$, the teacher will need to decide whether or not this is acceptable: arithmetic skills still play a key roll with this approach but the operations of addition, subtraction, and multiplication are no longer as important. No solutions of this type have been included.

It is important to stress the use of parentheses in this task. In the example above, $5 = (2 \times 3) - 1$, order of operations says that we should perform multiplication first and so, strictly speaking, adding the parentheses is not necessary. It is good practice, however, to add the parentheses as it helps stress the importance of order or operations and helps avoid errors in calculations. In a situation such as $4 = 2 \times (3 - 1)$, the parentheses become necessary in order to distinguish this from $5 = (2 \times 3) - 1$.

There is an interesting and challenging question which the teacher may wish to pose for students who are very comfortable with the arithmetic operations required for this task: what is the smallest whole number which can not be expressed in this way? The answer is 29 and the reasoning for this is quite interesting: students can check that all numbers from 1 through 28 are possible. To see that 29 is not possible, note that it is a prime number so cannot arise as a product of two numbers unless one of the factors is 1. It cannot be that one of the factors is 1 because the other three numbers are not big enough to make 29. This means that the last operation performed to get 29 must be addition or subtraction. Again, the other three numbers will not be big enough to make this work.

Teachers may wish to use a different set of numbers rather than 1, 2, 3, and 4. For example, using only three numbers (such as 1, 2, 3 or 2, 3, 4) cuts down significantly on the number of possibilities. On the other hand, adding numbers (working with 1, 2, 3, 4, and 5 for example) makes for an extremely challenging problem.
a. Using the fact that $9 = 3 \times 3$ we have

$$9 = 3 \times (4 - 1).$$

Also, using the fact that $9 = 8 + 1$, we have

$$9 = (4 \times 2) + 1.$$

b. Using the fact that $7 = 6 + 1$ we have

$$7 = (3 \times 2) + 1.$$

Also, using the fact that $7 = 8 - 1$, we have

$$7 = (4 \times 2) - 1.$$

Or, avoiding multiplication, we have

$$7 = 4 + 2 + 1.$$

c. Using the fact that $11 = 8 + 3$ we have

$$11 = (4 \times 2) + 3.$$

Also, using the fact that $11 = 12 - 1$ we have

$$11 = (4 \times 3) - 1.$$

d. We have $26 = 2 \times 13$ so if we can write $13$ using $1$, $3$, and $4$ we can get $26$ by doubling. We have $3 \times 4 = 12$ and $12 + 1 = 13$. Putting all of this together gives

$$26 = 2 \times ((3 \times 4) + 1).$$

Note that double parentheses are used here because there are three operations. The first operation is inside the innermost parentheses, $3 \times 4$. The next operation is in single parentheses, adding $1$. The final operation is not in parentheses, multiplying by $2$. 