F-LE Boiling Water

Alignments to Content Standards: F-LE.A.2 F-LE.A.1

Task

Below is a table showing the approximate boiling point of water at different elevations:

Elevation (meters above sea level)	Boiling Point (degrees Celsius)
0	100
500	98.2
1000	96.5
1500	94.7
2000	93.1
2500	91.3

a. Based on the table, if we were to model the relationship between the elevation and the boiling point of water, would a linear function be appropriate? Explain why or why not.

b. Below are some additional values for the boiling point of water at higher elevations:

Elevation (meters above sea level)	Boiling Point (degrees Celsius)
5,000	83.2
6,000	80.3

7,000	77.2
8,000	74.3
9,000	71.5

Based on the table, if we were to model the relationship between the elevation and the boiling point of water, would a linear function be appropriate? Explain why or why not.

c. When the information from both tables is combined, would a linear function be appropriate to model this data? What kind of function would you use to model the data? Why?

IM Commentary

This task examines linear models for the boiling point of water as a function of elevation. Two sets of data are provided and each is modeled quite well by a linear function. The slopes of the two linear models are different, however, and so a single linear function is nc as accurate when the two sets of data are considered simultaneously. It turns out that for the entire data set, an exponential model fits the data better than a linear model. One of tl reasons linear models are so important is that they are relatively easy to construct and evaluate and even non-linear phenomena like the boiling point of water can still be modeled over suitable domains by (several) linear functions.

We can see in several ways that the boiling point of water cannot be a linear function of elevation for a (sufficiently) large domain. If the boiling point were to continue to decrease at a constant rate as we gain elevation, then it would eventually become less than absolute zero which does not make any sense. Also, from a scientific point of view, a definition of th boiling point of water is the temperature at which ''the vapour pressure of the liquid is equal to the pressure exerted on the liquid by the surrounding environmental pressure'' (taken from http://en.wikipedia.org/wiki/Boiling). This means that the variation of the boilir point of water is tied to the variation of air pressure which happens to be modeled by an exponential function.

The data for this task was taken from here: http://www.engineeringtoolbox.com/boiling-points-water-altitude-d_1344.html. A Desmos sketch of this can be found here:

You can fit a line to each set of values independently and also see an exponential curve tha fits all the data better than a line can. Students should be encouraged to use the technolog so that they can visualize the calculations provided in the solution.

This task examines some aspects of MP4, Model with Mathematics. Students will use their knowledge of linear functions to decide whether or not a given set of data appears to adequately modeled by a linear function. Since the data is not exactly linear, students neec to interpret and quantify what this means. Classes with some experience in students also have a host of relevant statistical questions at their disposal, beginning with finding lines o best fit.

This task was designed for an NSF supported summer program for teachers and undergraduate students held at the University of New Mexico from July 29 through August 2, 2013 (http://www.math.unm.edu/mctp/). The participants suggested using the piecewise linear model for approximating the function.

Edit this solution

Solution

a. Notice that for the first set of data, the intervals for the elevation are all 500 meters. A linear function changes by equal amounts over equal intervals so we can begin by taking successive differences of the boiling points and see how they vary:

Elevation (meters above sea level)	Boiling Point (degrees Celsius)	Change in boiling point
0	100	--
500	98.2	-1.8
1000	96.5	-1.7
1500	94.7	-1.8
2000	93.1	-1.6
2500	91.3	-1.8

Note that there is no entry for the temperature change at 0 elevation because we have no point of comparison. We can see that the data is not exactly linear: there is a variation in th change in boiling point from -1.6 degrees Celsius per 500 meters to -1.8 degrees Celsius pe 500 meters. Some of this could be attributable to roundoff error. A linear function with a slope near -1.7 degrees Celsius per 500 meters will provide a good model for this data, capturing the general trend with a good level of precision.

b. We make the same calculations with the second set of data here:

Elevation (meters above sea level)	Boiling Point (degrees Celsius)	Change in boiling point
5,000	83.2	--
6,000	80.3	-2.9
7,000	77.2	-3.1
8,000	74.3	-2.9
9,000	71.5	-2.8

Like in part (a) the data is not linear but the change in boiling point is pretty regular. The boiling point change per 1000 meters is between-2.8 degrees Celsius per 1000 meters and

3.1 degrees per 1000 meters. A linear function with a slope close to -2.9 degrees Celsius pe 1000 meters would provide a good model for this data.

c. When we combine the two sets of data we can see that the variation in the boiling point for the lower elevations was near -1.7 degrees Celsius per 500 meters. This is the same as - 3.4 degrees Celsius per 1000 meters, substantially more than the -2.9 degrees Celsius per 1000 meters which we found in (b). A linear model over the elevation range from 0 to 900C meters would capture the general trend of a decreasing boiling point as elevation increase It would miss an important feature of the relationship, however, namely the fact that the change in boiling point relative to elevation appears to depend on the elevation. More specifically, the magnitude of the change in boiling point is getting smaller as the elevation increases. This makes sense as the boiling point cannot continue to go down indefinitely a the same rate as we go up in elevation.

We can apply this analysis to the interval between 2500 meters from the first table and 50(meters from the second table. The linear model from part (a) would predict a decrease in the boiling point of about 8.5 degrees. The linear model from part (b) would predict a decrease of about 7.3 degrees. The actual decrease according to the tables is 8.1 degrees. This makes sense: the rate of decrease from 2500 meters to 5000 meters is slower than from 0 meters to 2500 meters but faster than from 5000 meters to 9000 meters.

For modeling this data, we could use a piecewise linear function, putting together the two linear approximations we get for the two sets of data. For example, we could use a slope o about -1.7 degrees Celsius per 500 meters for the first set of data and a slope of about -2.9 degrees Celsius per 1000 meters for the second set of data. These two lines meet for an elevation value in between 2500 meters and 5000 meters. We could use the first line until the two lines meet and then switch to the second line. Alternatively, to capture the fact tha the rate of decrease in the boiling point goes down as the elevation increases, we could us an exponential function to model the entire set of data. According to the desmos sketch provided in the commentary, the exponential function does a more accurate job of modeling the data but for practical purposes, the two linear functions give relatively accurate values and have the advantage of being easier to calculate or estimate by hand.