4.0A Multiples of 3, 6, and 7

Alignments to Content Standards: 4.OA.C  4.OA.B

Task

a. Make a list of the first ten multiples of 3.

b. Which of the numbers in your list are multiples of 6? What pattern do you see in where the multiples of 6 appear in the list?

c. Which numbers in the list are multiples of 7? Can you predict when multiples of 7 will appear in the list of multiples of 3? Explain your reasoning.

IM Commentary

This task investigates divisibility properties for the numbers 3, 6, and 7. Students first make a list of multiples of 3 and then investigate this list further, looking for multiples of 6 and 7. In addition to noticing that every other multiple of 3 is a multiple of 6, students will see that all multiples of 6 are also multiples of 3 because 3 is a factor of 6. Because the list of multiples of 3 is only long enough to show one multiple of 7, students will have to either continue the list or generalize based upon their observations from part (b). Unlike 6, there is no factor of 3 in 7 and so not every multiple of 7 has a factor of 3: in order to be a multiple of both 3 and 7, a number must be a multiple of 21.

One important difference in the multiples of 6 and 7 that appear in the list of multiples of 3 is that every multiple of 6 is also a multiple of 3. So 6, 12, 18, … all appear in the list of multiples of 3. Since 3 is not a factor of 7, not every multiple of 7 occurs in the list of multiples of 3. The teacher may wish to direct or ask the students about this key difference in the multiples of 6 and 7 which are also multiples of 3. The first solution
also refers to the fact that an odd number times an odd number is odd and the teacher may wish to go into this in greater depth as it is another good example of a pattern exemplifying 4.OA.5.

The Standards for Mathematical Practice focus on the nature of the learning experiences by attending to the thinking processes and habits of mind that students need to develop in order to attain a deep and flexible understanding of mathematics. Certain tasks lend themselves to the demonstration of specific practices by students. The practices that are observable during exploration of a task depend on how instruction unfolds in the classroom. While it is possible that tasks may be connected to several practices, only one practice connection will be discussed in depth. Possible secondary practice connections may be discussed but not in the same degree of detail.

This particular task helps illustrate Mathematical Practice Standard 8, Look for and express regularity in repeated reasoning. Fourth graders make their list of multiples of 3. Then they look for patterns and connections to the multiples of 6 and 7 as stated in the commentary. They purposely look for patterns/similarities, make conjectures about these patterns/similarities, consider generalities and limitations, and make connections about their ideas (MP.8). Students notice the repetition of patterns to more deeply understand relationships between multiples of 3 and multiples of 6. They can then compare this relationship to the relationship between multiples of 3 and multiples of 7 and look at the differences between the two sets of multiples. By examining the repeated multiples students can make conjectures and start to form generalizations. As they begin to explain their processes to one another, they construct, critique, and compare arguments (MP.3). Students would benefit from having access to \( \frac{1}{4} \)-inch graph paper and colored pencils for this task. The first solution shows some pictures that students could easily generate with those tools.

**Solutions**

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**Solution: 1 Pictures**

a. The first ten multiples of 3 are listed below:

\[3, 6, 9, 12, 15, 18, 21, 24, 27, 30.\]

b. The multiples of 6 in the list are highlighted in larger, bold face:

It appears as if every other number in the sequence is a multiple of 6. In order to see why, here is a picture showing \(10 \times 3\):

Notice that 2 groups of three make 1 group of six. This can be seen in the picture as 1 group of three purple squares and 1 group of three white squares.

So with an even number of threes, we can group them in pairs to make sixes. When there is an odd number of threes, there are some groups of six with a leftover group of three: in the picture, an odd number of threes leaves a purple group which does not match up with a white group (or vice versa).

c. The only number in the list that is a multiple of 7 is 21 which is \(7 \times 3\). If we write the list of multiples of 7:

\[
7, 14, 21, \\
28, 35, 42, \\
49, 56, 63, \\
70, 77, 84
\]

and then extend the list of multiples of 3:

\[
3, 6, 9, 12, 15, 18, 21, \\
24, 27, 30, 33, 36, 39, 42, \\
45, 48, 51, 54, 57, 60, 63, \\
66, 69, 72, 75, 78, 81, 84
\]
we can see that the first four multiples of 7 that appear in the list of multiples of 3 are 21, 42, 63, and 84.

21 is $3 \times 7$.

We got 42 as a multiple of 7 because $42 = 6 \times 7$. We can rewrite it as follows:

$$6 \times 7 = (2 \times 3) \times 7 = 2 \times (3 \times 7) = 2 \times 21$$

This is the same as 2 groups of 21. The next one they have in common is 63, which came from $9 \times 7$. As before, we can see that this is a multiple of 21:

$$9 \times 7 = (3 \times 3) \times 7 = 3 \times (3 \times 7) = 3 \times 21$$

In general, the multiples of 7 that appear in the list of multiples of 3 are also multiples of 21, and these happen each 7th multiple of 3 because each seven groups of 3 make a multiple of 7.
a. The first ten multiples of 3 are listed below:

\[3, 6, 9, 12, 15, 18, 21, 24, 27, 30.\]

b. The multiples of 6 in the list are highlighted in larger, bold face:

\[3, 6, 9, 12, 15, 18, 21, 24, 27, 30.\]

It appears as if every other number in the sequence is a multiple of 6. In order to see if this will continue, note that the multiples of 3 could also be written as

\[1 \times 3, 2 \times 3, 3 \times 3, 4 \times 3, 5 \times 3, 6 \times 3, 7 \times 3, 8 \times 3, 9 \times 3, 10 \times 3.\]

The even numbers, 2, 4, 6, … all have a factor of 2 and when this is multiplied by 3 the product has a factor of 6. This explains why the even numbered elements in the sequence are multiples of 6.

Alternatively, using 10 \times 3 as an example, we can write

\[
10 \times 3 = (5 \times 2) \times 3 \\
= 5 \times (2 \times 3) \\
= 5 \times 6
\]

so that 10 \times 3 is written as a multiple of 6. The second equation uses the associative property of multiplication. This argument works for any even number in place of 8 because each even number has a factor of 2.

On the other hand, an odd number times an odd number is odd so the 1st, 3rd, 5th, … elements of this sequence are odd: since 6 is a multiple of 2, any multiple of 6 is also a multiple of 2 and so must be even. This explains why the odd numbered elements of the sequence are not multiples of 6.

c. The only number in the list of multiples of 3 which is also a multiple of 7 is 21 = 3 \times 7. This is the seventh number in the sequence. We might guess that just as every second number in the sequence is a multiple of 2 so every seventh number in the sequence is a multiple of 7. We can check that this is so by writing equations like in part (b). We use 28 = 4 \times 7 as an example
28 \times 3 = (4 \times 7) \times 3 \\
= 4 \times (7 \times 3) \\
= 4 \times 21

This reasoning will show that every seventh number in the sequence is a multiple of 21.