Illustrative Mathematics

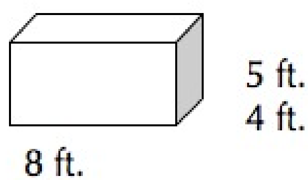
5.MD Cari's Aquarium

Alignments to Content Standards: 5.MD.C.5.b

Task

Cari is the lead architect for the city’s new aquarium. All of the tanks in the aquarium will be rectangular prisms where the side lengths are whole numbers.

a. Cari’s first tank is 4 feet wide, 8 feet long and 5 feet high. How many cubic feet of water can her tank hold?



b. Cari knows that a certain species of fish needs at least 240 cubic feet of water in their tank. Create 3 separate tanks that hold exactly 240 cubic feet of water. (*Ex: She could design a tank that is 10 feet wide, 4 feet long and 6 feet in height*.)

c. In the back of the aquarium, Cari realizes that the ceiling is only 10 feet high. She needs to create a tank that can hold exactly 100 cubic feet of water. Name one way that she could build a tank that is not taller than 10 feet.

IM Commentary

This task supports the standard 5.MD.5.b in that it asks students to use the volume formula and conceptual understanding to solve real-world problems. Please note, the standard calls for students to find the volume, but not necessarily a missing length.

Therefore, part a is aligned directly to the standard whereas parts b and c are extension problems. For part b, students might use the same numbers (10 feet wide, 4 feet long, 6 feet high) but order them differently (for example, 4 feet wide, 6 feet long, 10 feet high). This uses the commutative property of multiplication: it is not, however, a different shape of aquarium but rather the same shape oriented differently.

In addition, the task supports a natural extension of the standard 4.OA.4 ''Find all factor pairs for a whole number in the range 1–100.'' In parts b and c of this problem students are factoring a number larger than 100 and they are factoring into a product of three numbers as is appropriate for the 5th grade. If the teacher wishes to emphasize this aspect of the task, it could be useful to have a discussion about the tables provided in the solution. Because there are so many different possibilities, much care is needed to make sure to get a complete list without duplication: in the list for part (b) this is accomplished by putting in the largest possible length and then the largest possible width, slowly working down through all of the possibilities. For part (c) the numbers are listed a little differently but the same principle holds: the largest possible dimensions are used first.

This task also supports students thinking about what is appropriate in a real-life context. For example, a student could suggest in part b that the tank has a 1 foot by 1 foot base and is 240 feet tall. This would satisfy the constraints of the problem, but is highly unlikely in a real-life situation. In addition, many students might assume that part c is limiting them to working with a tank that has a height of 10 feet. Yet, in this context they could reason that a tank with a height of 5 feet would also work; just as long as Cari does not try to build a tank higher than the ceiling. Teachers should facilitate this task by asking guiding questions such as, “so tell me in your own words how your tank would look” or “if the ceiling is 10 feet high, how does this change how Cari might think about this last tank?”

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Solution

a. The tank can hold 160 cubic feet of water because:

$$\text{Length} \times \text{Width} \times \text{Height} = \text{Volume}$$
$$8 \text{ feet} \times 4 \text{ feet} \times 5 \text{ feet} = 160 \text{ feet}^3$$

Alternatively, the base of the tank is:

$$4 \text{ feet} \times 8 \text{ feet} = 32 \text{ feet}^2$$

Therefore,

$$\text{Area of Base} \times \text{ Height} = \text{Volume}$$
$$32 \text{ feet}^2 \times 5 \text{ feet} = 160 \text{ feet}^3$$

b. There are many possible solutions to this task. The whole number factor combinations are listed below. Please note that each set of factors is listed only once, though a student could reassign which factor represents length, width or height as long as she or he uses the same three factors.

Length	Width	Height	Volume
240 ft.	1 ft.	1 ft.	240 ft^3
120 ft.	2 ft.	1 ft.	240 ft^3
80 ft.	3 ft.	1 ft.	240 ft^3
60 ft.	4 ft.	1 ft.	240 ft^3
60 ft.	2 ft.	2 ft.	240 ft^3
48 ft.	5 ft.	1 ft.	240 ft^3
40 ft.	6 ft.	1 ft.	240 ft^3
40 ft.	3 ft.	2 ft.	240 ft^3
30 ft.	4 ft.	1 ft.	240 ft^3
30 ft.	2 ft.	2 ft.	240 ft^3
24 ft.	10 ft.	1 ft.	240 ft^3
24 ft.	5 ft.	2 ft.	240 ft^3
20 ft.	12 ft.	1 ft.	240 ft^3

20 ft.	6 ft.	2 ft.	240 ft^3
20 ft.	4 ft.	3 ft.	240 ft^3
16 ft.	15 ft.	1 ft.	240 ft^3
16 ft.	5 ft.	3 ft.	240 ft^3
15 ft.	4 ft.	4 ft.	240 ft^3
15 ft.	8 ft.	2 ft.	240 ft^3
12 ft.	5 ft.	4 ft.	240 ft^3
12 ft.	10 ft.	2 ft.	240 ft^3
10 ft.	6 ft.	4 ft.	240 ft^3
10 ft.	8 ft.	3 ft.	240 ft^3
8 ft.	6 ft.	5 ft.	240 ft^3

Again, the teacher should prompt students to think about what their tank would look like in real-life. A tank that is 40 feet long, 3 feet wide and 2 feet high would be very long, skinny and short. This tank would not be good for bigger fish, but it could be useful to watch something small that does not need a lot of space to move, such as a turtle.

c. There are many possible solutions to this task. All whole number factor combinations are listed below to ensure that students understand that they cannot simply choose any three factors that multiply to 100 $ft.^3$ without considering the restrictions on height. Make sure that students think through the real-world implications of this problem: Cari's tank would work as long as it not higher than 10 feet tall.

Length	Width	Height	Volume
100 ft.	1 ft.	1 ft.	100 $ft.^3$
1 ft.	100 ft.	1 ft.	100 $ft.^3$

50 ft.	2 ft.	1 ft.	100 $ft.^3$
2 ft.	50 ft.	1 ft.	100 $ft.^3$
50 ft.	1 ft.	2 ft.	100 $ft.^3$
50 ft.	2 ft.	1 ft.	100 $ft.^3$
25 ft.	4 ft.	1 ft.	100 $ft.^3$
25 ft.	1 ft.	4 ft.	100 $ft.^3$
1 ft.	25 ft.	4 ft.	100 $ft.^3$
4 ft.	25 ft.	1 ft.	100 $ft.^3$
25 ft.	2 ft.	2 ft.	100 $ft.^3$
2 ft.	25 ft.	2 ft.	100 $ft.^3$
20 ft.	5 ft.	1 ft.	100 $ft.^3$
5 ft.	20 ft.	1 ft.	100 $ft.^3$
10 ft.	1 ft.	10 ft.	100 $ft.^3$
1 ft.	10 ft.	10 ft.	100 $ft.^3$
10 ft.	10 ft.	1 ft.	100 $ft.^3$
10 ft.	5 ft.	2 ft.	100 $ft.^3$
10 ft.	2 ft.	5 ft.	100 $ft.^3$
5 ft.	2 ft.	10 ft.	100 $ft.^3$
5 ft.	10 ft.	2 ft.	100 $ft.^3$
2 ft.	10 ft.	5 ft.	100 $ft.^3$

2 ft.	5 ft.	10 ft.	100 $ft.^3$
5 ft.	4 ft.	5 ft.	100 $ft.^3$
4 ft.	5 ft.	5 ft.	100 $ft.^3$
5 ft.	5 ft.	4 ft.	100 $ft.^3$