8.NS Placing a square root on the number line

Alignments to Content Standards: 8.NS.A.2

Task

Place \(\sqrt{28}\) on a number line, accurate to one decimal point.

IM Commentary

The purpose of the task is to make connections between the definition and properties of squares and square roots and ordering on the number line, as prescribed by standard 8.NS.2. The key algebraic underpinning to this task is that squaring and square-rooting preserve order, i.e., if \(a\) and \(b\) are positive, then

\[ a < b \quad \text{if and only if} \quad a^2 < b^2 \]

The solution to this task requires repeatedly applying this idea.

At the teacher’s discretion a calculator with basic functions may be appropriate, though we note that all that is really required by the solution is three 2-digit multiplication problems. In fact, students might be shocked that they can complete the task without a calculator! As such, teachers might consider allowing students to work in small groups to brainstorm strategies for placing \(\sqrt{28}\) on the number line.

Solution
We repeatedly use the fact that for positive numbers $a$ and $b$, whenever $a < b$ we also have $a^2 < b^2$, and vice versa (both of which can be seen by comparing areas of squares with side lengths $a$ and $b$).

Because 28 is between 25 and 36, we know its square root must be between 5 and 6. To get a better approximation, we can square numbers between 5 and 6 and see how they compare to 28. Since 28 is closer to 25 than to 36, we’ll start by squaring numbers closer to 5. First, we find $5.1^2 = 26.01$, which is less than 28, so $\sqrt{28} > 5.1$. Next, we find $5.2^2 = 27.04$, so we similarly conclude that $\sqrt{28} > 5.2$. Finally, after computing $5.3^2 = 28.09$, we see that $\sqrt{28} < 5.3$, and so $\sqrt{28}$ is between 5.2 and 5.3.

The decimal expansion of $\sqrt{28}$ thus begins 5.2…, though we note that by the last calculation above, $\sqrt{28}$ is much closer to 5.3. In fact, the expansion continues

$$\sqrt{28} = 5.291502…$$