Illustrative Mathematics

S-MD Fred's Fun Factory

Alignments to Content Standards: S-MD.A.2 S-MD.B.7 S-MD.B.5

Task

A famous arcade in a seaside resort town consists of many different games of skill and chance. In order to play a popular “spinning wheel” game at Fred's Fun Factory Arcade, a player is required to pay a small, fixed amount of 25 cents each time he/she wants to make the wheel spin. When the wheel stops, the player is awarded tickets based on where the wheel stops -- and these tickets are then redeemable for prizes at a redemption center within the arcade.

Note: this particular game has no skill component; each spin of the wheel is a random event, and the result from each spin of the wheel is independent of the results of previous spins.

The wheel awards tickets with the following probabilities:

1 ticket	35%
2 tickets	20%
3 tickets	20%
5 tickets	10%
10 tickets	10%
25 tickets	4%

100 tickets	1%

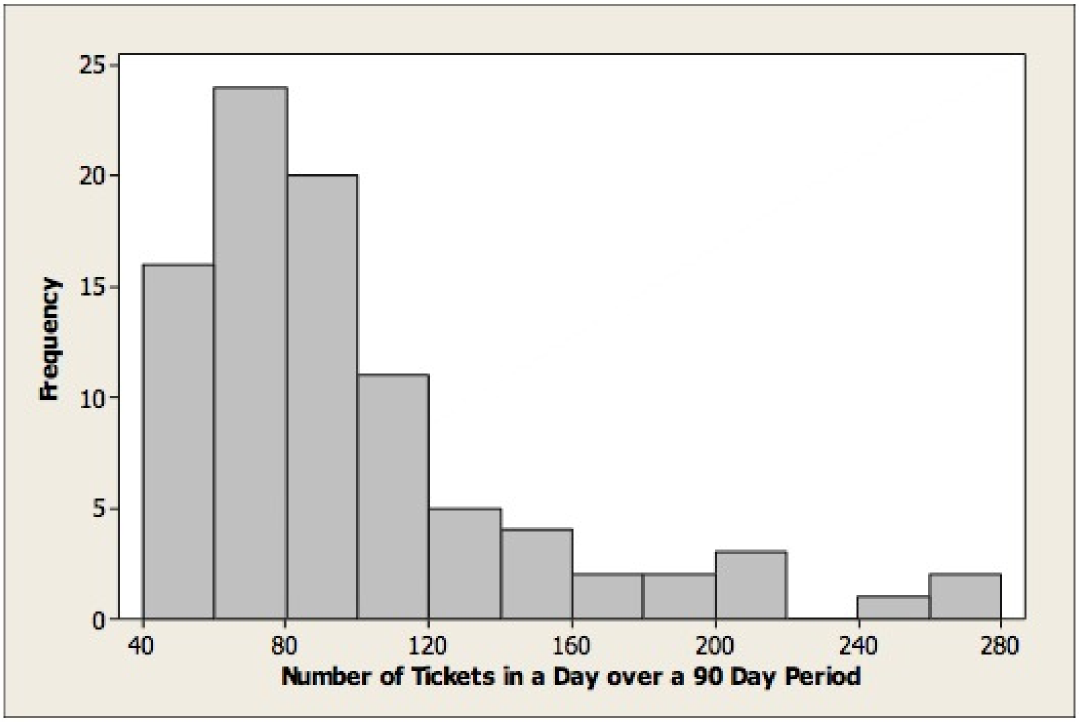
(Note: A picture of a wheel fitting these parameters is included.)

a. If a player were to play this game many, many times, what is the expected (average) number of tickets that the player would win from each spin?

b. The arcade often provides quarters to its customers in $5.00 rolls. Every day over the summer, a young boy obtains one of these quarter rolls and uses all of the quarters for the spinning wheel game. In the long run, what is the average number of tickets that this boy can expect to win each day using this strategy?

c. One of the redemption center prizes that the young boy is playing for is a trendy item that costs 300 tickets. It is also available at a store down the street for $4.99. Without factoring in any enjoyment gained from playing the game or from visiting the arcade, from a strictly monetary point of view, would you advise the boy to try and obtain this item based on arcade ticket winnings or to just go and buy the item at the store? Explain.

d. The histogram below summarizes the results of 90 summer days of the boy playing the game using his "$5 roll per day" strategy. The first bar in the histogram represents those days where 40 to 59 tickets were won.



i. For approximately what percentage of the 90 days did the boy earn fewer than 100 tickets in a day?

ii. For approximately what percentage of the 90 days did the boy earn 200 or more tickets in a day?

iii. For approximately what percentage of the 90 days did the boy earn 300 or more tickets in a day?

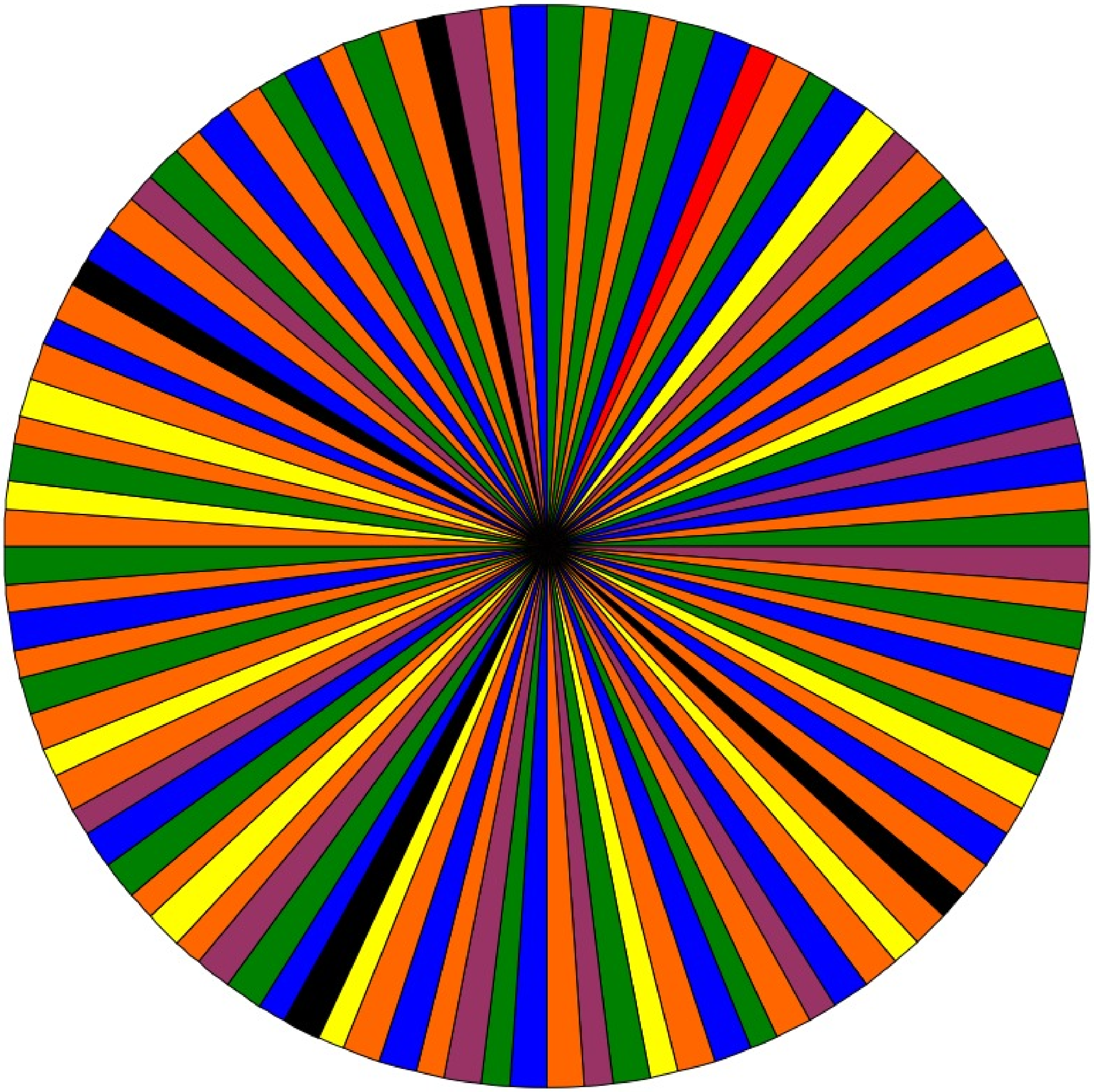
e. To maintain the spinning wheel game machine, the arcade manager adopts a strategy of emptying the money box (that's where the quarters go after they are inserted in the machine) each time she refills the machine with a new roll of tickets. The ticket refill rolls contain 5000 tickets, and the machine is designed to hold $1300 in quarters in its money box. Assume that the machine was fully loaded with 5000 tickets and had an empty money box when it was first used. Using the manager's maintenance strategy, is there any chance that the money box could become completely full with quarters or overflow with quarters? Explain.

Arcade Wheel

Color	Tickets
Orange	1

Green	2
Blue	3
Yellow	5
Purple	10
Black	25
Red	100

Graphics Note: The wheel diagram was developed in Microsoft Excel using its "Pie Chart" graph building feature. The intent is that each wedge represents 1% of the pie (3.6 degrees). There are 35 orange wedges (each representing a win of 1 ticket) to correspond to the 35% probability of obtaining 1 ticket in a spin, 20 green wedges (each representing a win of 2 tickets) to correspond to the 20% probability of obtaining 2 tickets in a spin, and so on.



IM Commentary

Developing the probability distribution of $X =$ the number of tickets earned over a set of 20 plays is an extremely daunting exercise. However, determining the average number of tickets earned over a set of 20 plays (in this case $E(x)$) is accessible and can be approached using the formulas and concepts of expected value. Students can then use this knowledge to apply decision making in context; and via a simulation/sample of 90 cases, students can also have a fairly respectable sense as to the general commonness and rarity of possible values for X.

Edit this solution

Solution

a. $1 \cdot .35 + 2 \cdot .2 + 3 \cdot .2 + 5 \cdot .1 + 10 \cdot .1 + 25 \cdot .04 + 100 \cdot .01 = 4.85 \text{ tickets}$. Note: If for some reason a student wanted to develop a simulation of several game plays to approximate this expected value (as opposed to using the formula), the student should simulate a number of spins, and the average number of tickets won per game should be very close to this number.

b. $5 roll of quarters = 20 quarters, so $20 \cdot 4.85 = 97$ tickets each day, i.e., each set of 20 spins.

c. Various answers are possible, but they should all deal with the idea that based on an expected average of 4.85 tickets for each game play, we expect it will require about 62 game plays (61.86) on average to earn 300 tickets. 62 games * 25 cents = $15.50, more than three as much as buying at the store. Other less formal arguments could use estimates or values obtained from questions (a) and (b) above. For example, if the average is about 5 tickets per spin, a person needs about 60 spins on average to hit 300 tickets. 60 spins is $15 which is 3 times more than $4.99, and so on. Or, with an average of about 100 tickets a day (from question (b)), that means that 3 days would be needed on average. That's 3 rolls of quarters = $15 to get the 300 tickets, etc.

d. i. $\frac{60}{90} = 66.7$% (close approximations are acceptable, but the histogram is designed such that 60 cases fit the criteria out of a total of 90 cases).

ii. $\frac{6}{90} = 6.7$% (see note above)

iii. $\frac{0}{90} = 0$%

e. No. Various arguments can be made, but the central theme is that even if the game returns the minimum of only 1 ticket per play over 5000 plays (extremely unlikely), that represents only 5000 quarters = $1250 which is less than the $1300 capacity. Or from another perspective, the money box can hold 5200 quarters ($\frac{1300}{.25} = 5200$), and the most quarters that the machine can earn from a 5000 ticket roll would be 5000 quarters (again, under the extremely unlikely minimal case of 5000 games of 1 ticket each).