

**S-CP Lucky Envelopes**

Alignments to Content Standards:  S-CP.A.3

**Task**

There are four red envelopes, four blue envelopes, and four $1 bills, which will be placed in four of the eight envelopes. Define the event $A$ as “you pick a lucky envelope (one that has a $1 bill in it)” and event $B$ as “you pick a blue envelope”.

a. Suppose one $1 bill is placed in a blue envelope, and the three remaining $1 bills are placed in three red envelopes.

i. If you choose one envelope at random, what is the probability that you pick a lucky envelope? How would you write this probability symbolically (using letters $A$ and/or $B$)?

ii. If you know that the envelope you picked is blue, what is the probability that you picked a lucky envelope? How would you write this probability symbolically (using letters $A$ and/or $B$)?

iii. Did knowing that the envelope is blue change the probability of getting a lucky envelope?

b. Now suppose we redistributed the four $1 bills between two blue and two red envelopes.

i. If you choose one envelope at random, what is the probability that you pick a lucky envelope? How would you write this probability symbolically (using letters $A$ and/or $B$)?

ii. If you know that the envelope you picked is blue, what is the probability that
you picked a lucky envelope? How would you write this probability symbolically (using letters $A$ and/or $B$)?

iii. Did knowing that the envelope is blue change the probability of getting a lucky envelope?

c. Two events are **independent** if knowing that one event has occurred has no effect on the probability that the other has occurred.

i. Are the events $A$ and $B$ from part (a) independent events?

ii. Are the events $A$ and $B$ from part (b) independent events?

iii. Suppose two events $E$ and $F$ are independent. What does the definition of independence imply about the two probabilities $P(E)$ and $P(E|F)$?

**IM Commentary**

This task builds on students’ prior knowledge and understanding of conditional probability, and introduces the concept of independence of events.

This task would be easy to illustrate by using real envelopes in two different colors, and $1 bills. In part (c.iii) of the problem we expect students to recognize that if two events $E$ and $F$ are independent, then $P(E) = P(E|F)$. Confirming that these two probabilities are equal is a way of determining that two events are independent. Teachers should also mention that it is also true that if $E$ and $F$ are independent events, then $P(F) = P(F|E)$.

**Solution**

a. i. Out of 8 envelopes, 4 have $1 bills in them. So the probability of picking a lucky envelope (with a $1 bill) is \( \frac{4}{8} = \frac{1}{2} \). Symbolically we write this as $P(A) = \frac{1}{2}$.

ii. In this part, we only consider blue envelopes. Out of 4 blue envelopes, only one has a $1 bill in it. So the probability of picking the lucky envelope is $\frac{1}{4}$. This is a
conditional probability: the probability that the envelope is lucky given that the envelope is blue. Symbolically we write this as $P(A|B) = \frac{1}{4}$.

iii. Yes, knowing that the envelope picked was blue changed the probability that the envelope is a lucky envelope. It decreased the probability of picking a lucky envelope from $\frac{1}{2}$ to $\frac{1}{4}$.

b. i. Out of 8 envelopes, 4 have $1 bills in them. So the probability of picking a lucky envelope (with a $1 bill) is $\frac{4}{8} = \frac{1}{2}$. Symbolically we write this as $P(A) = \frac{1}{2}$.

ii. In this part, we only consider blue envelopes. Out of 4 blue envelopes, two have $1 bills in them. So the probability of picking a lucky envelope is $\frac{2}{4} = \frac{1}{2}$. This is a conditional probability: the probability that the envelope is lucky given that the envelope is blue. Symbolically we write this as $P(A|B) = \frac{1}{2}$.

iii. No, knowing that the envelope picked was blue did not change the probability that the envelope is a lucky envelope. Either way, the probability of getting a lucky envelope is $\frac{1}{2}$.

c. i. In part (a), knowing that the envelope was blue (event $B$) changed the probability that the envelope was a lucky envelope (event $B$) from $\frac{1}{2}$ to $\frac{1}{4}$. Therefore, $A$ and $B$ are not independent events.

ii. In part (b), knowing that the envelope was blue (event $B$) did not change the probability that the envelope was a lucky envelope (event $B$). Therefore, $A$ and $B$ are independent events.

iii. If the events $E$ and $F$ are independent, the definition of independence implies that $P(E) = P(E|F)$. 