A-CED, A-REI Basketball

Alignments to Content Standards: A-REI.A.2 A-CED.A.1

Task

Chase and his brother like to play basketball. About a month ago they decided to keep track of how many games they have each won. As of today, Chase has won 18 out of the 30 games against his brother.

a. How many games would Chase have to win in a row in order to have a 75% wining record?

b. How many games would Chase have to win in a row in order to have a 90% winning record?

c. Is Chase able to reach a 100% winning record? Explain why or why not.

d. Suppose that after reaching a winning record of 90% in part (b), Chase had a losing streak. How many games in a row would Chase have to lose in order to drop down to a winning record below 55% again?

IM Commentary

This task provides a simple but interesting and realistic context in which students are led to set up a rational equation (and a rational inequality) in one variable, and then solve that equation/inequality for an unknown variable. It seems likely to be direct and relevant enough to be used for assessment purposes, either in part or in whole. Alternatively, this task could be used as a motivation for studying equations of this form in general, as while students might be able to solve the first part by trial and error, this becomes rather tedious for the later parts. Teachers might also find this task could be used to illustrate standard A-REI.A.1 if some more emphasis were placed on the reasoning behind the algebraic manipulations provided in the solutions.

Edit this solution

Solution

a. Currently, Chase has a winning record of 60%:

$$\frac{18 \text{ games won}}{30 \text{ games played}} = 0.6.$$

Clearly Chase has to win some more games to raise his win percent to 75%. Then in order to determine a winning record of 75% we need to add x amount of games to 18, and x amount of games to 30 and set this equal to 0.75:

$$\begin{aligned} \frac{18+x}{30+x} &= 0.75 \\ 18+x &= 0.75(30+x) \\ 18+x &= 22.5+0.75x \\ 0.25x &= 4.5 \\ x &= 18. \end{aligned}$$

So Chase will bring his win percent up to 75% if he wins the next 18 games.

b. The mechanics of this part are identical to the previous, replacing the desired 0.75 win record with 0.9:

$$\begin{aligned} \frac{18+x}{30+x} &= 0.9 \\ 18+x &= 0.9(30+x) \\ 18+x &= 27+0.9x \\ 0.1x &= 9 \\ x &= 90. \end{aligned}$$

So Chase will bring his win percent up to 90% if he wins the next 90 games.

c. No, because in order to have a 100% winning record, Chase will have needed to win every game. We can see this is not the case because in the scenario he won 18 out of 30, in other words; he had already lost 12 out of 30, and cannot change this fact no matter how many more games he plays. It's interesting to see where the algebra from the previous two parts breaks down: If x were the number of games Chase could win to get to a 100% winning record, we would have

$$\begin{aligned} \frac{18+x}{30+x} &= 1 \\ 18+x &= 30+x \\ 18 &= 30, \end{aligned}$$

which is clearly false.

d. After reaching a 90% winning record by winning 90 consecutive games, Chase has won 108 out of 120 games. If he were to lose x consecutive games from this point, we would have a record of 108 wins out of $120+x$ game. For this win percent to be less than 0.55, we solve:

$$\begin{aligned} \frac{108}{120+x} &< 0.55 \\ 108 &< 0.55(120+x) \\ 108 &< 60+0.55x \\ 42 &< 0.55x \\ 76.36 &< x. \end{aligned}$$

So Chase will need to lose the next 77 games in order for his win percentage to drop below 55%.