F-BF Flu on Campus

Alignments to Content Standards: F-BF.A.1.c

Task

Suppose the swine flu, influenza H1N1, is spreading on a school campus. The following table shows the number of students, n, that have the flu d days after the initial outbreak. The number of students who have the flu is a function of the number of days, $n = f(d)$.

d **(days)**	0	2	6	8	12	16	24
$n = f(d)$ **(number of students infected)**	3	9	16	30	55	45	32

There is a school store on campus. As the number of students who have the flu increases, the number of tissue boxes, b, sold at the school store also increases. The number of tissue boxes sold on a given day is a function of the number of students who have the flu, $b = g(n)$, on that day.

n **(number of students infected)**	0	3	8	9	12	16	18	30	32	38	45	50	55
$b = g(n)$ **(number of tissue boxes sold)**	1	4	8	12	13	18	24	33	34	40	45	51	57

a. Find g(f(0)) and state the meaning of this value in the context of the flu epidemic. Include units in your answer.

b. Fill in the chart below using the fact that $b = g(f(d))$.

d **(days)**	0	2	6	8	12	16	24
b **(number of tissue boxes sold)**							

c. For each of the following expressions, explain its meaning in the context of the problem, and if possible, give an approximation of its value. Justify your answer.

i. $g(f(16))$

ii. $g(f(18))$

iii. $f(g(9))$

IM Commentary

The purpose of this problem is to have students compose functions using tables of values only. Students are asked to consider the meaning of the composition of functions to solidify the concept that the domain of g contains the range of f. Part c requires two non-trivial pieces of reasoning: First, in (ii), students are asked to reason out an approximate value for $g(f(18))$, even though this number cannot be directly evaluated from the given data. The solution provides a reasonable but fairly inexact procedure for doing this -- a more precise approach would be to attempt to interpolate the functions with precise formulas, but most attempts to do this are too simplistic (e.g., a linear model) or beyond the scope of the task (e.g., a logistic model). Second, in (iii), students are presented with the expression $f(g(9))$, which can be evaluated when f and g are considered as abstract functions, but which is meaningless when considered in the context of the problem. The distinction between these two points of view is subtle, but important -- the "jumping" in and out of the real-world context is an important aspect of mathematical modeling. Regardless, students should certainly come away with the message that it may be possible to compose a pair of functions in one direction but not the other.

Submitted by Becky Hall to the Fourth illustrative Mathematics task writing contest.

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Solution

a. Since $f(0)$ represents the number of students who have the flu 0 days after the outbreak began, we see from the table that $f(0) = 3$ students. So, $g(f(0)) = g(3) = 4$ tissue boxes. Thus, the number of tissue boxes sold on the day of the outbreak was 4 boxes.

b.

d	0	2	6	8	12	16	24
b	4	12	18	33	57	45	34

c. i. The table constructed in part (b) represents exactly the function $g(f(d))$, so we can read off the value precisely: $g(f(16)) = 45$, representing the number of tissue boxes sold 16 days after the initial outbreak.

ii. Similarly, $g(f(18))$ represents the number of tissue boxes sold 18 days after the initial outbreak. However, since $f(18)$ cannot be directly read off of the first table, we cannot use the tables to determine this value precisely. We can instead reason approximately as follows: Between $d = 16$ and $d = 24$, we see a decline of about 1-2 infected students per day, so a reasonable approximation would be that two days after $d = 16$, at $d = 18$, we drop to about 42 infected students. Similarly, from the second table, it appears that with 42 infected students, it is reasonable to predict sales of about 42 or 43 tissue boxes.

iii. As abstract functions being represented by the tables above, we can read from the tables that $g(9) = 12$ and $f(12) = 55$, and hence we can compute that $f(g(9)) = f(12) = 55$. However, this value is meaningless in the context of the problem. Indeed, consider the units on the relevant quantities: Since $g(9) = 12$ is measured in tissue boxes (the number of tissue boxes sold when there were 9 students infected with the flu), and the contextual domain of f is measured in a number of students, it does not make sense to input 12 tissue boxes into the function f.