F-IF Domains

Alignments to Content Standards: F-IF.A.1

Task

For the functions in (a)–(f),

a. List the algebraic operations in order of evaluation. What restrictions does each operation place on the domain of the function?

b. Give the function's domain.

a. $y = \frac{2}{x-3}$

b. $y = \sqrt{x-5} + 1$

c. $y = 4 - (x-3)^2$

d. $y = \frac{7}{4-(x-3)^2}$

e. $y = 4 - (x-3)^{1/2}$

f. $y = \frac{7}{4-(x-3)^{1/2}}$

IM Commentary

The purpose of this task to help students think about an expression for a function as built up out of simple operations on the variable, and understand the domain in terms

of values for which each operation is invalid (e.g., dividing by zero or taking the square root of a negative number).

It is understood that unless otherwise specified, the domain of a function given by an algebraic expression is the set of values of the variable for which the expression is defined. The task's solution is writen in the context of the domain being a subset of the real numbers. The instructor should make this unwritten assumption explicit if being used in a setting where this is ambiguous. Alternatively, in a more advanced setting, the task coule be used to touch on complex number stadnards like N-CN.5 by exploring how the domains would differ in the complex setting.

Edit this solution

Solution

a. i. In order of evaluation:

i. Operation: Subtract 3 from x. This does not restrict the domain, since we can subtract 3 from any number.

ii. Operation: Divide 2 by the above result. This means that $x - 3$ can't equal 0, so $x \neq 3$.

ii. The domain is $x \neq 3$.

b. i. In order of evaluation:

i. Operation: Subtract 5 from x. This does not restrict the domain, since we can subtract 5 from any number.

ii. Operation: Take the square root of the above result. This means that $x - 5$ can't be negative, so $x \geq 5$.

iii. Operation: Add 1 to the above result. This does not restrict the domain, since we can add 1 to any number.

ii. The domain is $x \geq 5$.

c. i. In order of evaluation:

i. Operation: Subtract 3 from x. This does not restrict the domain, since we can subtract 3 from any number.

ii. Operation: Square the above result. This does not restrict the domain, since we can square any number.

iii. Operation: Subtract the above result from 4. This does not restrict the domain, since we can subtract any number from 4.

ii. The domain is all real numbers.

d. i. In order of evaluation:

i. Operation: Subtract 3 from x. This does not restrict the domain, since we can subtract 3 from any number.

ii. Operation: Square the above result. This does not restrict the domain, since we can square any number.

iii. Operation: Subtract the above result from 4. This does not restrict the domain, since we can subtract any number from 4.

iv. Operation: Divide 7 by the above result. This means that the result of step C can't equal 0, so

$$(x-3)^2 \neq 4.$$

Thus,

$$\begin{aligned} x-3 &\neq 2 \\ x &\neq 5 \\ \text{and} \quad x-3 &\neq -2 \\ x &\neq 1. \end{aligned}$$

ii. The domain is all x except $x = 1, 5$.

e. i. In order of evaluation:

i. Operation: Subtract 3 from x. This operation does not restrict the domain, since we can subtract 3 from any number.

ii. Operation: Raise the above result to the power of $1/2$. This means that $x-3$ can't be negative, so $x \geq 3$.

iii. Operation: Subtract the above result from 4. This operation does not restrict the domain, since we can subtract any number from 4.

ii. The domain is $x \geq 3$.

f. i. In order of evaluation:

i. Operation: Subtract 3 from x. This operation does not restrict the domain, since we can subtract 3 from any number.

ii. Operation: Raise the above result to the power of $1/2$. This means $x - 3$ can't be negative, so $x \geq 3$.

iii. Operation: Subtract the above result from 4. This operation does not restrict the domain, since we can subtract any number from 4.

iv. Divide 7 by the above result. This means that the result of step C can't equal 0, so

$$(x-3)^{1/2} - 4 \neq 0$$
$$(x-3)^{1/2} \neq 4.$$

Thus,

$$x - 3 \neq 16$$
$$x \neq 19.$$

ii. The domain is $x \geq 3$, $x \neq 19$ -- that is, all numbers greater than or equal to 3 except 19.