F-IF Your Father

Alignments to Content Standards: **F-IF.A.1**

Task

a. Let F assign to each student in your math class his/her biological father. Explain why F is a function.

b. Describe conditions on the class that would have to be true in order for F to have an inverse.

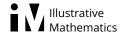c. In a case from part (b) in which F does not have an inverse, can you modify the domain so that it does?

IM Commentary

This is a simple task touching on two key points of functions. First, there is the idea that not all functions have real numbers as domain and range values. Second, the task addresses the issue of when a function admits an inverse, and the process of "restricting the domain" in order to achieve an invertible function. This theme repeats itself importantly in the algebraic setting -- see, for example, the task F-BF Graphs of Compositions.

Edit this solution
Solution

a. F is a function because it assigns to each student in the class exactly one element, his/her biological father.

b. An inverse of F would associate to a father of someone in the classroom any student to whom he is a father. If that father has multiple children in the classroom, this inverse relationship does not describe a function (for a relationship to be a function, there can be only one output associated to any given input). So a succinct condition for F to have an inverse is that no father of a student in the classroom is the father of more than one student, i.e., there are no siblings in the classroom.

c. As in the previous part, if there are siblings in the class, then there are two students in the class with the same father, preventing F from having an inverse. If we take any subset of the class which does not contain a pair of siblings, then the function F restricted to this domain has an inverse -- for example, we could take the domain to be the set of all oldest children in the classroom, or simply choose a collection of students with exactly one sibling from each family. With this domain, F will have an inverse function because the biological fathers in the range each have exactly one student in the class.