8.F Chicken and Steak, Variation 2

Alignments to Content Standards: 8.F.B.4

Task

Consider the relationship between the number of pounds of chicken and the number of pounds of steak that can be purchased for a barbecue on a fixed budget.

a. Write an equation for this relationship given that chicken costs $1.29 per pound, steak costs $3.49 per pound, and the total allotment for both is $100.

b. Sketch a graph for the equation. Describe how the amount of steak that can be purchased depends on the amount of chicken purchased. What is the significance of the graph's x- and y-intercepts?

c. Re-write the equation to show the amount of steak purchased in terms of the amount of chicken purchased. Use this equation to describe how the amount of steak purchased changes with each pound of chicken purchased.

d. Both the equation and the graph can be used to find amounts of chicken and steak that can be purchased for the barbecue on a fixed budget of $100. Explain how important features of the graph can be seen or found with the equation and describe what they say about the relationship between chicken and steak purchases.

IM Commentary

This task is intended strictly for instructional purposes with the goal of building understandings of linear relationships within a meaningful and, hopefully, somewhat

familiar context. The expectation for part (a) is that students simply translate the situation to equation form and not necessarily express the relationship as a linear function in slope-intercept form. Parts (b) and (c) align with content standards 8.F.5 and 8.F.4 respectively. Students are asked to analyze a graph of the relationship in part (b) and construct a linear function in part (c). Part (d) was added to advance understandings of graphic and symbolic representations, specifically with regard to how they model the same situation.

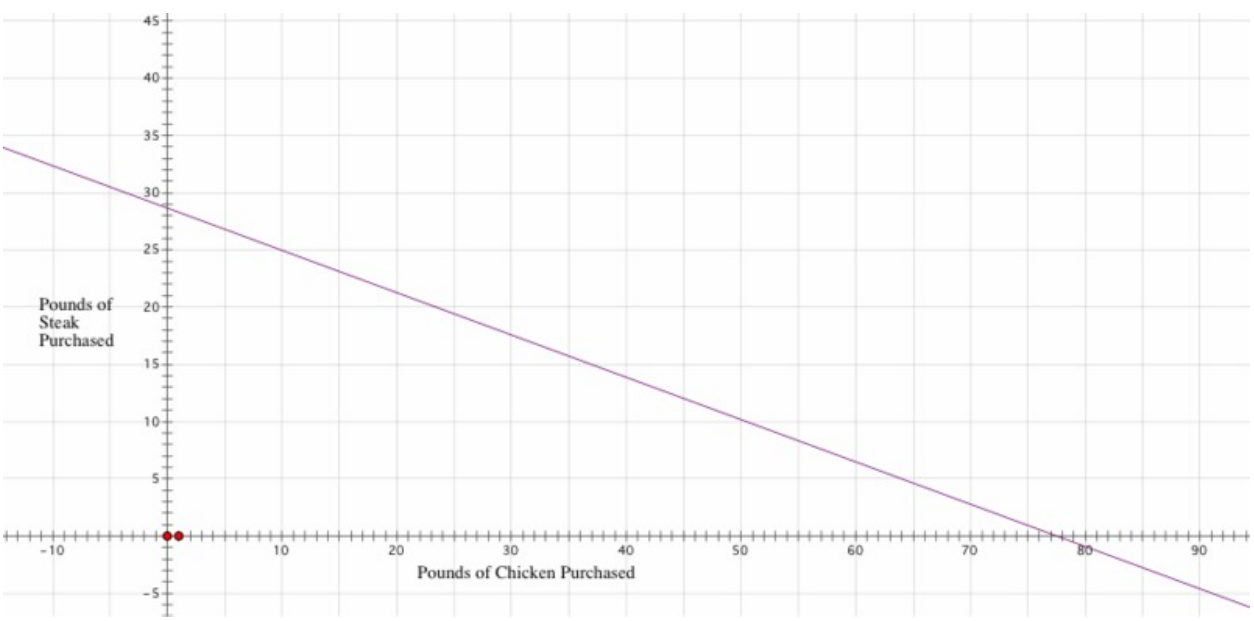
Some scaffolding may be needed for eighth graders to complete the task successfully, particularly with regard to initial interpretations of the graph and transforming the equation into slope-intercept form. The use of cooperative learning structures would greatly enhance student reasoning applied to the task, as would a carefully structured class discussion of small group findings. Under these conditions the Standards for Mathematical Practice 3 (construct viable arguments and critique the reasoning of others) and 4 (model with mathematics) would most certainly come into play.

Edit this solution

Solution

a. Let x represent the number of pounds of chicken purchased and y represent the number of pounds of steak purchased. Then

$$1.29x + 3.49y = 100$$



b. The graph shows that the amount of steak that can be purchased on a fixed budget decreases as the amount of chicken purchased increases. The x-intercept shows the maximum amount of chicken that can be purchased (77.5 lbs when no steak is purchased) and the y-intercept shows the maximum amount of steak that can be purchased (28.7 lbs when no chicken is purchased).

c. The equation from part (a) written in slope-intercept form for linear equations is

$$y = -\left(\frac{1.29}{3.49}\right)x + \frac{100}{3.49}$$

or

$$y = -0.37x + 28.65$$

with quotients rounded to the nearest hundredth. The x-coefficient reveals the rate of change in steak purchases per unit change in chicken purchases. Specifically, the amount of steak that can be purchased given a $100 budget starts at 28.65 lbs and drops by 0.37 lb (or a little over one-third of a pound) with each pound of chicken purchased.

d. A robust explanation of how the graph and equation both represent the same situation is expected here. At a minimum, students should recognize (1) how the rate of change is represented by the slope of the graph and by the x-coefficient in the equation, (2) that the constant term in the equation represents the y-intercept on the graph, and (3) how the equation can be manipulated to find the graph's x-intercept. All explanations should be grounded in the context of the situation represented by the graph and equation.