F-LE Algae Blooms

Alignments to Content Standards: F-LE.A.2 F-LE.A.4 F-LE.A.1.c

Task

Algal blooms routinely threaten the health of the Chesapeake Bay. Phosphate compounds supply a rich source of nutrients for the algae, *Prorocentrum minimum*, responsible for particularly harmful spring blooms known as mahogany tides. These compounds are found in fertilizers used by farmers and find their way into the Bay with run-offs resulting from rainstorms. Favorable conditions result in rapid algae growth ranging anywhere from 0.144 to 2.885 cell divisions per day. Algae concentrations are measured and reported in terms of cells per milliliter (cells/ml). Concentrations in excess of 3,000 cells/ml constitute a bloom.

a. Suppose that heavy spring rains followed by sunny days create conditions that support 1 cell division per day and that prior to the rains *Prorocentrum minimum* concentrations measured just 10 cells/ml. Write an equation for a function that models the relationship between the algae concentration and the number of days since the algae began to divide at the rate of 1 cell division per day.

b. Assuming this rate of cell divison is sustained for 10 days, present the resulting algae concentrations over that period in a table. Did these conditions result in a bloom?

c. If conditions support 2 cell divisions per day, when will these conditions result in a bloom?

d. Concentrations in excess of 200,000 cells/ml have been reported in the Bay. Assuming the same conditions as in (c), when will concentrations exceed 200,000 cells/ml?

IM Commentary

The problem statement describes a changing algae population as reported by the Maryland Department of Natural Resources. In part (a), students are expected to build an exponential function modeling algae concentration from the description given of the relationship between concentrations in cells/ml and days of rapid growth (F-LE.2). The intent of part (b) is for students to gain an appreciation for the exponential growth exhibited despite an apparently modest growth rate of 1 cell division per day. Two solutions to part (c) are given, one using natural logarithms and one using logarithms to base 2. The solution can also be estimated graphically if the task is being used to motivate the need to solve exponential equations.

As structured, the task is best used in settings with sufficient time given students to process and make sense of the situation presented.

Solutions

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Solution: 1. Using the natural logarithm

a. One cell division per day implies that the number of cell divisions in t days is $1 \frac{\text{cell division}}{\text{day}} \cdot t \text{ days} = t \text{ cell divisions}$. Since, the number of cells doubles with each cell division, their concentration doubles with each cell division. So, the concentration, $C(t)$, of algae cells per milliliter in t days is given by

$$C(t) = 10 \cdot 2^t$$

b. Concentrations of algae (rounded to the nearest whole number) over the 10-day period are given the following table:

Day	1	2	3	4	5	6	7	8	9	10
cells/ml	20	40	80	160	320	640	1280	2560	5120	10,240

Hence, an algae bloom resulted in a little less than nine days.

c. Two cell divisions per day implies that the number of cell divisions in t days is $2 \frac{\text{cell division}}{\text{day}} \cdot t \text{ days} = 2 \cdot t \text{ cell divisions}$. Again, since a cell division implies the number of cells doubles, and since when the number of cells doubles, the concentration doubles, we have

$$C(t) = 10 \times 2^{2t}$$

To determine when concentrations are high enough to constitute a bloom, solve the following equation for t:

$$3000 = 10 \cdot 2^{2t}$$

Applying logarithms to solve for t:

$$\ln 300 = 2t \cdot \ln 2$$

$$t = \frac{\ln 300}{2 \cdot \ln 2} \approx 4.1 \text{ days.}$$

Hence, an algae bloom is formed under these conditions in a little over 4 days.

Similar procedures are used to determine when concentrations exceed 200,000 cells/ml.

$$200,000 = 10 \times 2^{2t}$$

yields

$$t = \frac{\ln 20,000}{2 \times \ln 2} \approx 7.1 \text{ days.}$$

Hence, the algae bloom exceeds concentrations of 200,000 cells/ml in a little over 7 days.

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Solution: 2. Using logarithms to base 2.

Standard scientific calculators often only have buttons for the natural logarithm or then base 10 logarithm. The first solution to this problems uses the natural logarithm, and therefore relies on the property $\ln a^b = b \ln a$. If students have access to a way of calculating logarithms to base 2 directly (such as with a CAS or an online calculator with this capability), then the following more direct solution to part (c) might be appropriate.

(c) Two cell divisions per day implies that the number of cell divisions in t days is $2\frac{\text{cell division}}{\text{day}} \cdot t \text{ days} = 2 \cdot t \text{ cell divisions}$. Again, since a cell division implies the number of cells doubles, and since when the number of cells doubles, the concentration doubles, we have

$$C(t) = 10 \times 2^{2t}$$

To determine when concentrations are high enough to constitute a bloom, solve the following equation for t:

$$3000 = 10 \cdot 2^{2t}$$

Dividing both sides by 10 we get

$$2^{2t} = 300.$$

From the definition of logarithms, we have

$$2t = \log_2(300).$$

Typing "log_2 300" into google we get $\log_2(300) = 8.2288$, so

$$t = \frac{8.2288}{2} \approx 4.1 \text{ days}.$$

Hence, an algae bloom is formed under these conditions in a little over 4 days.

Similar procedures are used to determine when concentrations exceed 200,000 cells/ml.

$$200,000 = 10 \times 2^{2t}$$

yields

$$t = \frac{\log_2 20,000}{2} \approx 7.1 \text{ days.}$$

Hence, the algae bloom exceeds concentrations of 200,000 cells/ml in a little over 7 days.