F-BF Temperatures in degrees Fahrenheit and Celsius

Alignments to Content Standards: F-LE.A.2 F-BF.B.4.a

Task

Let f be the function that assigns to a temperature in degrees Celsius its equivalent in degrees Fahrenheit.

a. The freezing point of water in degrees Celsius is 0 while in degrees Fahrenheit it is 32. The boiling point of water is 100 degrees Celsius and 212 degrees Fahrenheit. Given that the function f is linear, use this information to find an equation for f.

b. Find the inverse of the function f and explain its meaning in terms of temperature conversions.

c. Is there a temperature which is the same in degrees Celsius and in degrees Fahrenheit? Explain how you know.

IM Commentary

Temperature conversions provide a rich source of linear functions which are encountered not only in science but also in our every day lives when we travel abroad. The first part of this task provides an opportunity to construct a linear function given two input-output pairs. The second part investigates the inverse of a linear function while the third part requires reasoning about quantities and/or solving a linear equation.

In part (c), students could also argue try to give an intuitive argument for the existence of such a point, reasoning via the linear relationship between degrees Fahrenheit (F) and degrees Celsius (C). Namely, we can first easily check that $F < C$ when, say, $C = -100$, and that $F > C$ when, for example, $C = 0$. Since there is a linear relationship between F and C, there must then be some value of C between $C = -100$ and $C = 0$ where in fact $F = C$. This is a subtle but important argument, one that eventually leads to more advanced topics like continuity and the Intermediate Value Theorem.

Solutions

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Solution: 1

a. Since f is a linear function of x, the temperature in degrees Celsius, we can write $f(x) = ax + b$. We are given that zero degrees Celsius converts to 32 degrees Fahrenheit so

$$32 = f(0) = b.$$

In order to find the slope, a, we can use the second piece of data given, namely that 100 degrees Celsius converts to 212 degrees Fahrenheit:

$$212 = f(100) = a(100) + 32.$$

Solving for a gives $a = \frac{9}{5}$. So to convert the temperature in degrees Celsius to degrees Fahrenheit, the appropriate linear function is

$$f(x) = \frac{9}{5}x + 32.$$

b. The function f multiplies the input, x, by $\frac{9}{5}$ and then adds 32. Let's call the inverse g. Then an equation describing g could first subtract 32 from its input and then divide by $\frac{9}{5}$. Dividing by $\frac{9}{5}$ is the same as multiplying by $\frac{5}{9}$ so we find

$$g(x) = \frac{5}{9}(x - 32)\,.$$

The function g takes as inputs the temperature in degrees Fahrenheit and gives as output the corresponding temperature in degrees Celsius.

c. If there is a temperature x which is the same in degrees Celsius and in degrees Fahrenheit then we would have

$$f(x) = x.$$

If we solve the equation $\frac{9}{5}x + 32 = x$ for x, we find that $x = -40$. So -40 is the temperature which registers the same on the Fahrenheit and Celsius scales. For temperatures *above* -40, the temperature in degrees Fahrenheit will be greater than the corresponding temperature in degrees Celsius while for temperatures *below* -40 the temperature in degrees Fahrenheit is less than the corresponding temperature in degrees Celsius.

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Solution: Alternate b)

Our function, f, described by the equation

$$y = \frac{9}{5}x + 32$$

takes a temperature in degrees Celsius and gives its equivalent in degrees Fahrenheit. If we wish to have a function that takes a temperature in degrees Fahrenheit and gives its equivalent in degrees Celsius we switch the roles of x and y in the equation above, and solve for x in terms of y. This means subtracting 32 from both sides of the equation and then multiplying both sides of the new equation by $\frac{5}{9}$, giving:

$$x = \frac{5}{9}(y - 32)\,.$$

We are used to y being the dependent variable and x being the independent variable so we can rewrite this, using function notation, as

$$g(x) = \frac{5}{9}(x - 32)$$

and this function converts the temperature in degrees Fahrenheit to the corresponding temperature in degrees Celsius.