7.EE Miles to Kilometers

Alignments to Content Standards: 7.EE.A

Task

The students in Mr. Sanchez's class are converting distances measured in miles to kilometers. To estimate the number of kilometers, Abby takes the number of miles, doubles it, then subtracts 20% of the result. Renato first divides the number of miles by 5, then multiplies the result by 8.

a. Write an algebraic expression for each method.

b. Use your answer to part (a) to decide if the two methods give the same answer.

IM Commentary

In this task students are asked to write two expressions from verbal descriptions and determine if they are equivalent. The expressions involve both percent and fractions. This task is most appropriate for a classroom discussion since the statement of the problem has some ambiguity.

The Standards for Mathematical Practice focus on the nature of the learning experiences by attending to the thinking processes and habits of mind that students need to develop in order to attain a deep and flexible understanding of mathematics. Certain tasks lend themselves to the demonstration of specific practices by students. The practices that are observable during exploration of a task depend on how instruction unfolds in the classroom. While it is possible that tasks may be connected to several practices, the commentary will spotlight one practice connection in depth. Possible secondary practice connections may be discussed but not in the same degree of detail.
This task helps illustrate Mathematical Practice Standard 2; where mathematically proficient students make sense of quantities and their relationships in problem situations, create coherent representations of the problem given and attend to the meaning of the quantities, not just how to compute them. Students will translate the description of the situation into algebraic equations, decontextualizing. Also, they will need to think about what quantities should be represented by variables and how those variables relate to each other. This will require them to “make sense of quantities and their relationships in problem situations.” The teacher might direct the students’ discussion by asking questions such as: “What do the numbers used in the task represent?” “What operations will we need to use to solve this task?” It is important that students just don’t know how to compute the numbers in the tasks, but are able to understand the meaning of the quantities and are flexible in the use of operations and their properties.

Adapted from Algebra: Form and Function, McCallum et al., Wiley 2010

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Solution

a. Abby’s method starts by doubling \(m\), giving \(2m\). She then takes 20% of the result, which we can write \(0.2(2m)\). Finally she subtracts this from \(2m\), giving

\[2m - (0.2)2m\]

Renato’s method starts by dividing \(m\) by 5, giving \(m \div 5 = \frac{m}{5}\), and then multiplies the result by 8, giving

\[8 \left( \frac{m}{5} \right)\]

b. Abby’s expression can be simplified as follows:

\[2m - (0.2)2m = 2m - 0.4m = (2 - 0.4)m = 1.6m.\]

(The step where we rewrite \(2m - 0.4m\) as \((2 - 0.4)\) uses the distributive property.)

Renato’s method gives

\[8 \cdot \frac{m}{5} = 8 \cdot \frac{1}{5} \cdot m = \frac{8}{5} \cdot m = 1.6m.\]
So the two methods give the same answer and the expressions are equivalent.