F-LE Basketball Rebounds

Alignments to Content Standards: F-LE.A.2 F-LE.A.1.c

Task

According to Wikipedia, the International Basketball Federation (FIBA) requires that a basketball bounce to a height of 1300 mm when dropped from a height of 1800 mm.

a. Suppose you drop a basketball and the ratio of each rebound height to the previous rebound height is 1300:1800. Let h be the function that assigns to n the rebound height of the ball (in mm) on the n^{th} bounce. Complete the chart below, rounding to the nearest mm

n	$h(n)$
0	1800
1	
2	
3	

b. Write an expression for $h(n)$.

c. Solve an equation to determine on which bounce the basketball will first have a height o less than 100 mm.

IM Commentary

The purpose of this task is to introduce students to an exponential decay function in the form $f(x) = ab^x$, with a concrete interpretation of the decay factor b. The context is designed to make clear the meaning of b, with each successive value marked by a clear physical phenomenon, the bounce, which leads to a reduction in height by a factor of b. Notice that the natural domain of the function is not all real numbers, but rather the set of positive integers. This can lead to a useful discussion about domains and modeling.

Two solutions to part (c) are shown: a solution students may use who have access to the tool, "take the ln of both sides," and a graphing solution for students who have not studied logarithms.

Teachers can extend this task by having students engage in a related classic classroom activity. Students bounce a basketball beneath a sonar detector (such as the Calculator Based Ranger from Texas Instruments). The CBR and graphing calculator will generate a graph of the ball's position at times throughout the bouncing. Students can trace along this graph to find at least five rebound heights and then compute the average of the $\frac{h(n)}{h(n-1)}$ ratios. Students can then use this average along with their knowledge of the initial height of the ball to develop an exponential model for the data.

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Solution

a. On the first rebound, the ball must rise to a height of 1300 mm according to the IBF regulations.

To determine the second rebound height, 1300 mm must be multiplied by the required fraction $\frac{1300}{1800} = \frac{13}{18}$:

$$1300\left(\frac{13}{18}\right) = \frac{16,900}{18} \approx 939 \text{ mm}$$

For each succeeding rebound the previous rebound height is multiplied by $\frac{13}{18}$;

n	$h(n)$
0	1800
1	1300

2	$1300\left(\frac{13}{18}\right) \approx 939$
3	$938.9\left(\frac{13}{18}\right) \approx 678$

n	$h(n)$
0	1800
1	$1300 = 1800\left(\frac{1300}{1800}\right)$
2	$938.9 \approx 1300\left(\frac{13}{18}\right) = 1300\left(\frac{13}{18}\right)\left(\frac{13}{18}\right) = 1800\left(\frac{13}{18}\right)^2$
3	$678.1 \approx 938.9\left(\frac{13}{18}\right) \approx 1800\left(\frac{13}{18}\right)^2 \cdot \left(\frac{13}{18}\right) = 1800\left(\frac{13}{18}\right)^3$
n	$1800\left(\frac{13}{18}\right)^n$

b. Generalizing from the table, we have $h(n) = 1800\left(\frac{13}{18}\right)^n$: this makes sense because by the nth rebound, we've multiplied by $\frac{13}{18}$, n times.

c. The rebound on which the height will be approximately 100 mm can be determined by solving for n given $h(n) = 100$:

$$100 = 1800\left(\frac{13}{18}\right)^n$$

or

$$\frac{1}{18} = \left(\frac{13}{18}\right)^n$$

We can approach this equation in several ways. Algebraically, we could apply properties of logarithms (namely, the property $\ln(a^b) = b\ln(a)$) to solve such an equation:

$$\ln\left(\frac{1}{18}\right) = \ln\left(\frac{13}{18}\right)^n$$
$$\ln\left(\frac{1}{18}\right) = n \cdot \ln\left(\frac{13}{18}\right)$$

Evaluating these logarithms in a calculator and then rounding yields the equation

$$-2.890 \approx n \cdot (-0.325)$$

which tells us

$$n \approx 8.892$$

So, the first time the rebound will not be at least 100 mm with be on the 9th rebound.

Alternatively, we can use appropriate technology to find an approximate solution by graphing. For example, once the solution is in the form

$$100 = 1800\left(\frac{13}{18}\right)^n$$

we can graph $y = 100$ and $y = 1800\left(\frac{13}{18}\right)^n$ on the same xy-plane, and use the graphing calculator's functionality to find the approximate point of intersection (8.882,100). Hence, $n \approx 8.882$ and the first time the rebound will not be at least 100 mm with be on the 9th rebound. (In the example graph below, click the point of intersection to see its coordinates