F-LE Interesting Interest Rates

Alignments to Content Standards: F-LE.A.1

Task

City Bank pays a simple interest rate of 3% per year, meaning that each year the balance increases by 3% of the initial deposit. National Bank pays an compound interest rate of 2.6% per year, compounded monthly, meaning that each month the balance increases by one twelfth of 2.6% of the previous month's balance.

a. Which bank will provide the largest balance if you plan to invest $10,000 for 10 years? For 15 years?

b. Write an expression for $C(y)$, the City Bank balance, y years after a deposit is left in the account. Write an expression for $N(m)$, the National Bank balance, m months after a deposit is left in the account.

c. Create a table of values indicating the balances in the two bank accounts from year 1 to year 15. For which years is City Bank a better investment, and for which years is National Bank a better investment?

IM Commentary

While somewhat realistic interest rates are presented in the task, the schemes for calculating interest earned as detailed in the problem statement are not representative of standard banking practices where interest is normally compounded on a daily basis.

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Solution

a. A \$10,000 deposit at City Bank earns \$300 interest each year. Hence the City Bank account balance is after 10 years is

$$\$300(10) + \$10,000 = \$13,000.$$

National Bank, on the other hand, pays a lower interest rate, but compounds the interest monthly. This means that the account balance grows by $\frac{2.6}{12}\%$ each month. Accordingly, after 10 years (120 months), the National Bank account balance is

$$\$10,000\left(1 + \frac{.026}{12}\right)^{120} \approx \$12,965.65.$$

So City Bank provides the larger balance at the end of 10 years. However, similar computations show that the City Bank balance after 15 years is \$14,500 while the National Bank balance is about \$14,763.57. Thus National Bank provides the larger balance after 15 years.

b. Generalizing from (a) we have

$$C(y) = 300y + 10,000$$

and

$$N(m) = 10,000\left(1 + \frac{.026}{12}\right)^{m}$$

c. Using the expresions from (b) we generate the table below which shows that for investments of 10 years or less, City Bank provides the larger balance, but for investments of 11 years or longer, National Bank provides the greater balance.

Length of investment	City Bank Balance	National Bank Balance
One Year (12 months)	\$10,300.00	\$10,263.12
Two Year	\$10,600.00	\$10,533.16
Three Year	\$10,900.00	\$10,810.31
Four Year	\$11,200.00	\$11,094.75
Five Year	\$11,500.00	\$11,386.68

Six Year	$11,800.00	$11,686.29
Seven Year	$12,100.00	$11,993.78
Eight Year	$12,400.00	$12,309.36
Nine Year	$12,700.00	$12,633.25
Ten Year	$13,000.00	$12,965.65
Eleven Year	$13,300.00	$13,306.81
Twelve Year	$13,600.00	$13,656.94
Thirteen Year	$13,900.00	$14,016.28
Fourteen Year	$14,200.00	$14,385.08
Fifteen Year	$14,500.00	$14,763.58