# A-SSE Quadrupling Leads to Halving

**Alignments to Content Standards:**   A-SSE.A.1

## Task

Give an explanation, in terms of the structure of the expression below, why it halves in value when $n$ is quadrupled:
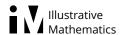
$$\frac{s}{\sqrt{n}}.$$

## IM Commentary

This question provides students with an opportunity to see expressions as constructed out of a sequence of operations: first taking the square root of $n$, then dividing the result of that operation into $s$. This way of looking at the expression helps the student see that

$$\frac{s}{\sqrt{4n}}$$

can be rewritten as

$$\frac{1}{2}\frac{s}{\sqrt{n}}.$$

Students studying statistics encounter the expression in this question as the standard deviation of a sampling distribution with samples of size $n$ when the distribution from

which the sample is taken has standard deviation $s$.

## Solution

The expression is a fraction in which the denominator is $\sqrt{n}$. The square root of $4n$ is twice the square root of $n$, because $\sqrt{4n} = \sqrt{4} \times \sqrt{n} = 2\sqrt{n}$. So quadrupling $n$ multiplies the denominator of the expression by $2$:

$$\frac{s}{\sqrt{4n}} = \frac{s}{\sqrt{4} \times \sqrt{n}} = \frac{s}{2 \times \sqrt{n}}.$$

Multiplying the denominator of a fraction by $2$ halves the value of the fraction:

$$\frac{s}{2 \times \sqrt{n}} = \frac{1}{2} \times \frac{s}{\sqrt{n}}.$$

So multiplying $n$ by $4$ multiplies the value of the expression by $1/2$.