8.G Downhill

Alignments to Content Standards: 8.F.B.4

Task

A car is traveling down a long, steep hill. The elevation, \( E \), above sea level (in feet) of the car when it is \( d \) miles from the top of the hill is given by \( E = 7500 - 250d \), where \( d \) can be any number from 0 to 6. Find the slope and intercepts of the graph of this function and explain what they mean in the context of the moving car.

IM Commentary

This task would be especially well-suited for instructional purposes. Students will benefit from a class discussion about the slope, \( y \)-intercept, \( x \)-intercept, and implications of the restricted domain for interpreting more precisely what the equation is modeling.

One potential confusion students may have follows from the subtle difference between what the car is doing and the idea of slope as the ratio between the change in vertical distance on the graph and the change in horizontal distance on the graph. Because the car is traveling one mile on a down-hill slope, the situation could be represented as a right triangle with a hypotenuse of 5,280 ft and a leg of 250 ft; using the Pythagorean Theorem they would find that the other leg is approximately 5,274 ft. Following through on this interpretation, a student might conclude that the car travels a horizontal distance of approximately 5,274 ft for every 250 ft in vertical distance and arrive at a slope of approximately -0.047. While this is, in some sense, the slope of the hill, it is not the slope of the function as described. This interpretation yields numbers that are very close to the situation described in the task, yet conceptually different since the distance traveled by the car would now be expressed in terms of horizontal distance traveled as opposed to distance along the slope of the hill to compute the
elevation. If students do indeed pursue this line of reasoning, the task provides an opportunity to compare and contrast the graph of the function and what it represents with a drawing of the hill and the vertical and horizontal distances traversed with each mile down the slope.

Edit this solution

Solution

The slope is $-250$, which can be interpreted as a rate of change:

The elevation will decrease by 250 feet for every mile it travels.

The $E$ intercept is 7500, which means the elevation of the car is 7500 feet above sea level at mile 0.

The $d$ intercept is 30, which means that if the car traveled in this same manner for 30 miles, it would be at an elevation of 0 (in other words, at sea level), but since this function is only valid for $0 \leq d \leq 6$, it doesn't really mean anything for this particular car.