6.RP Security Camera

Alignments to Content Standards: 6.RP.A.3

Task

A shop owner wants to prevent shoplifting. He decides to install a security camera on the ceiling of his shop. Below is a picture of the shop floor plan with a square grid. The camera can rotate 360°. The shop owner places the camera at point P, in the corner of the shop.

![Shop Floor Plan]

a. The plan shows where ten people are standing in the shop. They are labeled A, B, C,
D, E, F, G, H, J, K. Which people cannot be seen by the camera at P?

b. What percentage of the shop is hidden from the camera? Explain or show work.

c. The shopkeeper has to hang the camera at the corners of the grid. Show the best place for the camera so that it can see as much of the shop as possible. Explain how you know that this is the best place to put the camera.

**IM Commentary**

The last question has more than one answer, in the sense that there are three spots that could be considered "best." These three locations all cover the same amount of the store while at the same time miss less of the store than all other possible spots.

A more advanced version of the last question that removes the requirement for the camera to be at a corner of the grid would be appropriate at grade 8 when students are studying parallel lines. Stay tuned for this version of the task.

This task is based on a task developed by the MARS/Shell Centre team Mathematics Assessment Resource Service. The task is shared with the with attribution, non-commercial, share-alike Creative Commons License.

**Edit this solution**

**Solution**

a. With the camera at point P, shoppers F and H are hidden from the camera.

b. There are 20 squares on the grid. If a line is drawn from point P to point T and beyond, the region that is hidden from the camera has an area of 3 squares (this region is composed of a triangle with an area of 1 square and a rectangle with an area of 2 squares; see the figure below).
There are a total of 17 out of 20 squares visible from point P.

\[
\frac{17}{20} = 0.85, \text{ so 85\% of the store is visible, and 15\% of the store is hidden from point P.}
\]

c. Looking at the figure below, the best places to place the camera are Q, R, and S.

Case 1: Consider what is hidden from a camera at point Q. If a line is drawn from point Q through point U, it will intersect point W. The area that is hidden is a triangle with area 2 squares. The area visible to the camera at point Q is 18 out of 20 squares.

\[
\frac{18}{20} = 0.9 \text{ or 90\% of the store is visible, so 10\% of the store is hidden from point Q.}
\]

If a point is directly below or to the left of Q, then the area that is hidden will be entirely on the right-hand side of the floor plan. If you draw a line from any point that is to the left of Q through point U, the portion of the floor plan that will be hidden will be the
region below the line you just drew on the right side of the floor plan (see the figure below for one example).

![Diagram](image.png)

This region will always have a greater area than the region hidden from a camera at point Q, so Q will be a better location than any point to the left of it.

Case 2: Consider what is hidden from a camera at point R. If a line is drawn from point R through U, it will intersect point V.

![Diagram](image2.png)

The area that is hidden on the right side of the floor plan is a triangle with area 1 square. The same area is hidden on the left side of the store (because the floor plan is symmetrical and R is on the line of symmetry). So there area visible to the camera at point R is 18 out of 20 squares.
\[
\frac{18}{20} = 0.9 \text{ or } 90\% \text{ of the store is visible, so } 10\% \text{ of the store is hidden from point } R.
\]

What if we put a security camera at any point below R? The part of the floor plan that is hidden in this case will be the region of the floor plan below the lines connecting that point with points T and U. Since those regions will have an area that is greater than the area hidden from point R, point R is a better location for a security camera than any of the ones below R.

Case 3: From point S, exactly the same amount of the store is visible as from point Q above (because the floor plan is symmetrical and Q and S are mirror images). A camera at any point directly below or to the right of S will have a greater amount of the floor plan hidden, just as with all points to the left of point Q.

90\% of the store is visible to the camera when it is placed at any one of the points Q, R, or S; from any other area in the store, less than 90\% of the store is visible. Assuming the camera is located at the corners of the grid, the best spots to place the camera to prevent shoplifting are at points Q, R, or S.