N-Q Ice Cream Van

Alignments to Content Standards: N-Q.A.1

Task

You are considering driving an ice cream van during the summer vacation. Your friend, who “knows everything” tells you that “It’s easy money.” You make a few inquiries and find that the van costs $600 per week to rent. Each ice cream cone costs 50 cents to make and sells for $1.50.

For each of the questions below, show all work and include an explanation of your method of solution.

a. How many ice cream cones would you have to sell each week just to cover the cost of renting the van?

b. In order to sell the ice cream cones, you have a choice of driving the van through neighborhoods or parking the van in a public area. Typical selling data is that one can sell an average of 35 ice cream cones per hour at each of your planned stops if driving through neighborhoods, while you can sell an average of 30 ice cream cones per hour if one parks the van in a public area.

   i. If you choose to drive the van, you will have to consider the time spent driving the van, which will depend on the average speed from stop to stop on your route, as well as the cost of gasoline, which will depend on the number of miles per gallon the van gets. Make reasonable estimates for these and any other costs you feel would be relevant. If you drive an average of 180 miles per week, how many ice cream cones would you have to sell just to cover the cost of driving the van for a week (not including rental costs)?

   ii. If you choose to park the van, you will have to pay a one-time seasonal permit fee and weekly space rental. If the seasonal permit costs $90.00 and space rental ranges from $140 to $150 per week, how many ice cream cones would you have
to sell just to cover the cost of parking the van for a week (again, not including rental costs)? Identify any assumptions you make.

c. How many hours a week will you have to work in order to make this “easy money”? After how many hours would the amounts you earned under each of the two options be the same? How much money might you be able to make if you were willing to work really hard? Identify and take into account any additional expenses for the additional hours. Explain your reasoning clearly.

**IM Commentary**

The purpose of this task is to engage students, probably working in groups, in a substantial and open-ended modeling problem. Students will have to brainstorm or research several relevant quantities, and incorporate these values into their solutions.

The Standards for Mathematical Practice focus on the nature of the learning experiences by attending to the thinking processes and habits of mind that students need to develop in order to attain a deep and flexible understanding of mathematics. Certain tasks lend themselves to the demonstration of specific practices by students. The practices that are observable during exploration of a task depend on how instruction unfolds in the classroom. While it is possible that tasks may be connected to several practices, the commentary will spotlight one practice connection in depth. Possible secondary practice connections may be discussed but not in the same degree of detail.

This task helps illustrate Mathematical Practice Standard 4, Model with mathematics. There are multiple parts to this open-ended task involving the operation of a mobile ice cream cone business. There are ample opportunities for students to apply the mathematics they know as they develop a solution pathway in addition to making reasonable assumptions and approximations and revising these as part of the modeling cycle during the solution process. Certain assumptions that impact the business will need to be determined or approximated such as the current cost of gasoline, average speed of vehicle from stop to stop, average fuel consumption of the vehicle, and other possible operating costs. As students make their assumptions and approximations and determine their costs, hours, and profit, they must also be prepared to justify their solution pathway and evaluate the appropriateness of their model which supports the demonstration of Mathematical Practice Standard 3, Construct viable arguments and critiques the reasoning of others.
Solution

a. Calculations and explanation should indicate an understanding that the profit from the sale of one ice cream cone is $1.00 (selling price of $1.50 minus $0.50 to make each cone) meaning that 600 cones would have to be sold to cover the cost of hiring the van for the week.

b. i. The cost of gasoline and any other operating costs identified by the student should reasonably reflect current costs. Miles per gallon estimates should also be reasonable for a van—currently a range of 10-25 mpg might be considered reasonable. Additionally, an estimate of the average speed of the van while traveling from stop to stop should be reasonable -- probably 15-30 mph since most of the driving will be in residential neighborhoods. Calculations should reflect the number of gallons of gasoline used per week and the cost for that many gallons and answers should be rounded in appropriate ways. The solution should include the number of ice cream cones needed to be sold at $1.00 profit per cone to equal the dollar amount plus an additional cone to cover any cents.

For example:

Assume that gasoline costs $3.35 per gallon and that the van averages 15 miles per gallon. To drive 180 miles, the van will use 12 gallons of gas (180m/15mpg with care taken regarding units). Gasoline will cost approximately $40.20 for the week (12g x $3.35). At a profit of $1.00 per cone, 41 ice cream cones (40/1.00 rounded up since fractional cones cannot be sold and 40 cones will not cover the whole cost) would have to be sold to cover the weekly operating cost of the van. It is important that the student recognizes that at an assumed average speed of, say, 20mph, there will be 9 hours (180m/20mph with care taken regarding units) spent driving, during which time it is impossible to sell a cone.

ii. Since the permit cost covers the entire season, its cost will have to be averaged over the number of weeks that the ice cream is to be sold. That will require that a student make some reasonable assumption of the number of weeks in order to complete this calculation. The $90 should be divided by the assumed number of weeks and that amount added to the rental fee (any number within the given range of fees) to determine the cost per week. The solution should include the number of ice cream cones needed to be sold at $1.00 profit per cone to equal the dollar amount plus an additional cone to cover any cents.
For example:

If you want to sell ice cream for 12 weeks, the permit cost will be $7.50 per week. If the space can be rented for $145, parking the van will cost $152.50 per week. At a profit of $1.00 per cone, 153 ice cream cones would have to be sold to cover the weekly operating cost of the van.

c. For either method above, just to meet your costs you will need to make $600 to cover the van rental plus the costs computed in part 2.a or part 2.b. This total is divided by the profit per ice cream cone ($1.00) and then divided by 30 (in the case of parking) or 35 (in the case of driving) to determine the number of hours it will take each week to make this much. The analysis in the case of driving also needs to reflect the additional hours computed based on the average speed assumed in part 2.a.

For example:

The driving method will cost $600 + $40.20 = $640.20. This requires that 641 ice cream cones be sold each week. At 35 cones per hour it will take about 18.5 hours per week (641 cones/35 cones per hour rounded to the nearest half hour) to sell that many cones. You would need to work 27.5 hours per week just to cover your expenses for the van.
The parking method will cost $600 + $152.50 = $752.50. This requires that 753 ice cream cones be sold each week and you would need to work about 25 hours per week.

To make any money for yourself, you would have to work additional hours each week and perhaps work for more weeks. Students should select a reasonable number of working hours—perhaps between 35 and 65 hours—a reasonable number of working weeks—perhaps between 10 and 16—and compute profit for the number of selected hours and/or weeks. Solutions that include driving the van might allow for additional miles driven, although it will be important to recognize the additional "time premium". Solutions that include renting a parking space for the van might include additional weeks, slightly lowering the permit cost per week.

For example:

Assume that you plan to drive the van around, driving an average of 180 miles each week, and selling an average of 35 cones per hour. If you were willing to work 50 hours per week for 12 weeks, you would be able to cover your expenses in the first 27.5 hours you worked (these assumptions match the conditions for question 2.a so see calculations in the solution for 2.a above), meaning that the additional 22.5 hours generates money that you will be able to keep. During that 22.5 additional hours you would sell 22.5 x 35 = 787 additional cones at $1.00 profit per cone each week. You
would make $787 per week for 12 weeks or a total of $9444 for the 12 weeks of work.

Assume that you plan to rent a parking space for the van. The cost per week actually decreases slightly if you increase the number of weeks you rent the space because the permit fee is divided across more weeks. If you were willing to work 50 hours per week and work for 15 weeks, your cost per week for parking the van would be $6.00 for the permit plus $140 - $150 for space rental. If you can rent the space for $145, the cost for the week would be $600 + $151 = $751. You would have to sell 751 cones to cover your expenses. You would be able to sell 50 x 30 cones per week or 1500 cones per week so you would make $749 per week to keep for yourself (1500 – 751 cones at a profit of $1.00 per cone). If you worked for 15 weeks you would make $11,235.

In order to address the question of the number of hours spent before the amounts earned are equal, we should assume that the number of weeks worked would be the same under both options. So for example: under the assumption that you will work for 12 weeks, under the parking option you cover your costs in about 25 hours, and under the driving option in about 27.5 hours. During those 2.5 extra hours, the parking option generates $75 of extra profit. However, the driving option produces $5 of extra profit per hour, meaning that the parking option’s advantage will disappear after 15 hours. Therefore, the amounts earned under the two options will be equal at 42.5 hours.