6.G Wallpaper Decomposition

Task

Jamie is planning to cover a wall with red wallpaper. The dimensions of the wall are shown below:

a. How many square feet of wallpaper are required to cover the wall?

b. Wallpaper comes in long rectangular strips which are 24 inches wide. If Jamie lays the strips of wallpaper vertically, can she cover the wall without wasting any wallpaper? Explain.

c. If Jamie lays the strips of wallpaper horizontally, can she cover the wall without wasting any wallpaper? Explain.
**IM Commentary**

The purpose of this task is for students to experiment with composition and decomposition of polygons to examine shapes in a real world context. To find the area of the wall, students will decompose a pentagon into simpler shapes (for example, a rectangle and a triangle). In parts (b) and (c), students need to think about how they can cut rectangular strips of wallpaper in order to fit them on the wall. This involves decomposing and then (re)composing polygons. For parts (b) and (c) students will need to make an assumption about the top of the wall, namely that the perpendicular line in the picture bisects the base of the wall (as it appears to in the picture).

The problem indicates that the wallpaper is a solid color (red) in order to eliminate certain complications, namely that patterned wallpaper will not necessarily "match up" and look correct if strips are cut, rotated (or reflected), and then reassembled. There are only certain ways that patterned wallpaper can be pieced together and this could either be discussed in class or portions of this video can be used:

The teacher may wish to have students think about what might happen with patterned wallpaper as an extension of this problem.
Solution

a. One natural way to find the area of the wall is by decomposition. If we draw a horizontal line 8 feet up the wall, we break the wall into an 8 foot by 8 foot square and a triangle:

The base of the triangle is 8 feet and its height is 2 feet. The square has an area of 64 square feet and the triangle has an area of

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\frac{8 \text{ feet} \times 2 \text{ feet}}{2} = 8 \text{ square feet.}
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So the total area of the wall is 64 + 8 = 72 square feet.

Another way to find the area of the wall is to notice that we can draw an 8 foot by 10 foot rectangle around the wall. This rectangle has an area of 80 square feet. To get the wall, we remove two right triangles. Together, these make half of a 2 foot by 8 foot rectangle and so they have an area of 8 square feet. Hence, the area of the wall is 80 - 8 = 72 square feet.

b. Jamie will need to cut the strips since the top of the wall does not lay horizontally while the top of the vertical strips of wallpaper will lay horizontally. One nice way Jamie could cover the wall with vertical strips would be to make cut two 18 foot strips of paper. Each of
these strips then needs to be cut "diagonally," eight feet up one side and nine feet up the other side:

c. With horizontal strips of wallpaper, 4 strips, each 8 feet long, will cover the square part of the wall leaving the triangle at the top. Jamie can next take a 4 foot long strip of wallpaper, making a 2 foot by 4 foot rectangle. If this rectangle is cut diagonally, Jamie gets two right triangles measuring 2 feet by 4 feet and these can be placed at the top of the wall: