OCCULTATIONS OF PLANETS BY THE ECLIPSED MOON

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Introduction

In a previous article[1] (referred to as I in the following) we made an investigation of occultations of some bright stars by the eclipsed Moon. In the present paper this investigation is extended to the planets Mars, Jupiter, Saturn and Uranus.

Due to the brightness of Mars, Jupiter and Saturn, occultations of these planets by the eclipsed Moon and also close conjunctions are spectacular phenomena, especially if the eclipse is total and Jupiter is involved. Therefore, a probability exists that these phenomena have been recorded; such old observations may in principle be used for the study of the rotation of the Earth, fixing both the Ephemeris Time and the hour angle at the observation place. For this reason we traced these cases for the period −100 to +3000 AD. For the faint planet Uranus, however, we limited the study to the present time.

In the first part the results are given. In the second part a discussion is given on the mean frequency of the phenomenon and on the periodicities which govern its distribution in time.

Occultations of the Planets

Obviously, an occultation of a celestial object by the eclipsed Moon can only take place when the object is near its opposition with the Sun, while its absolute geocentric latitude has to be below a certain limit. Contrarily to the stars, however, a planet with a given orbital inclination has a variable latitude at opposition, depending on its position in orbit. In figure 1 the geocentric position of the bright planets and of some stars is given as a function of the opposition date and the longitude for the epoch 1950·0. Although these opposition dates and the longitudes are slowly changing with time, the position of the orbits with respect to the stars in figure 1 remains almost the same during centuries. In figure 1 the limiting latitudes of 1°·79 and 2°·35 have also been indicated, below which occultations by respectively the totally and the partially eclipsed Moon are possible[1]. From the figure it is clear that for Mars and Saturn these occultations are possible only in certain regions of the ecliptic, while

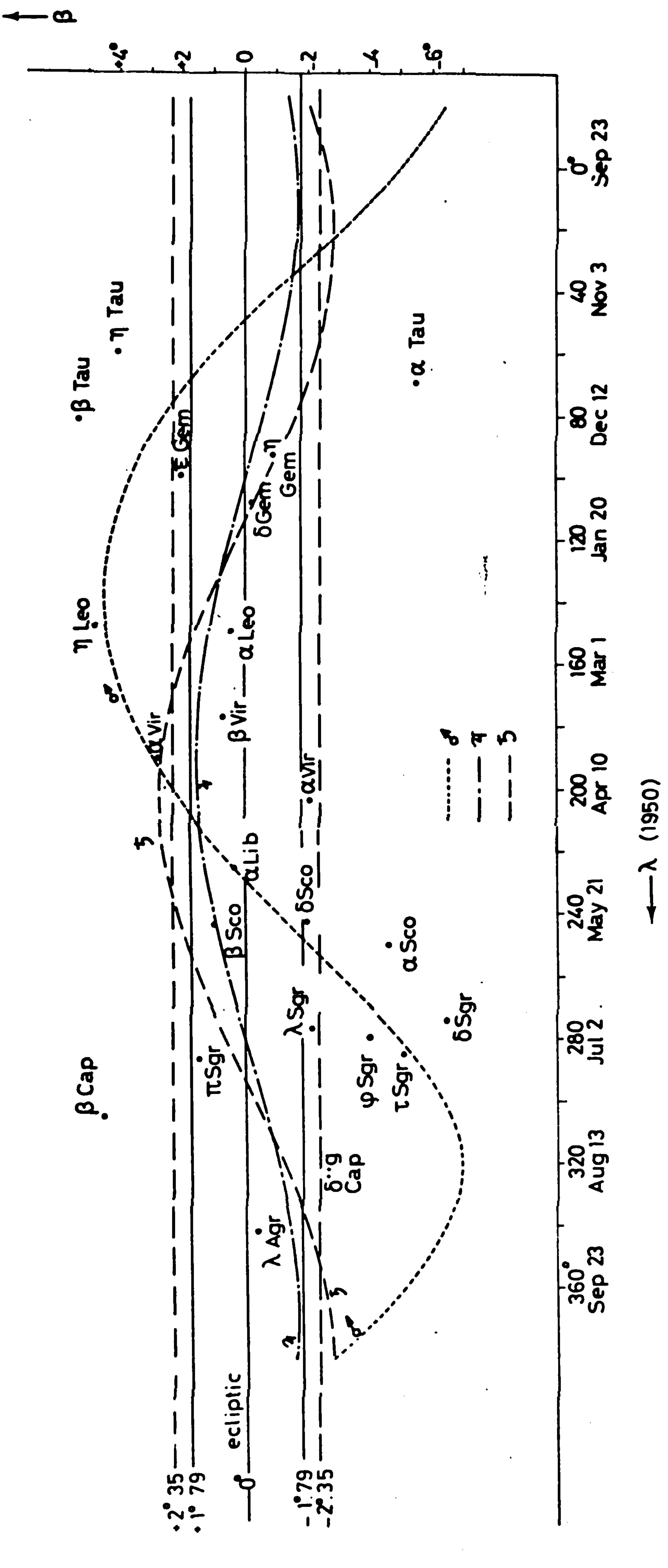

FIGURE 1. Geocentric positions of the planets at opposition as a function of the opposition date and the longitude for the epoch 1950·0. The limits of 1°·79 and 2°·35, below which occultations by respectively the totally and the partially eclipsed Moon are possible, are also indicated.

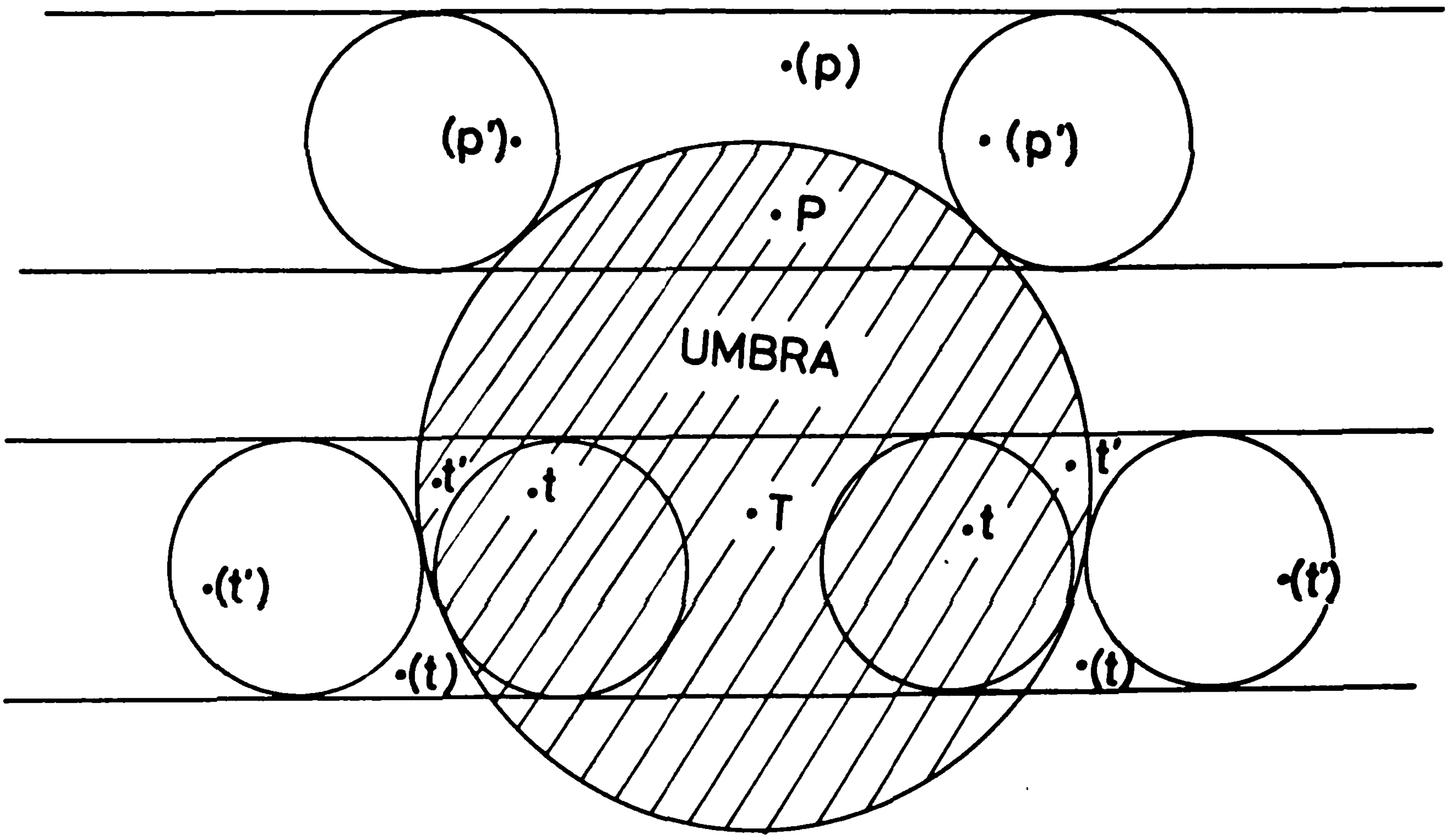

FIGURE 2. Path of the eclipsed Moon with respect to the occulted object for various types of occultations.

Jupiter may be occulted by the eclipsed Moon at any longitude. A further consequence of the variable latitude of the planets near opposition is, that this kind of occultation may be visible anywhere on Earth. This is contrary to the stars, where the fixed latitude determines the possible occultation region.

For the actual calculation of occultations of planets by the eclipsed Moon, we proceeded as follows. Firstly, a computer selection was made of conjunctions in longitude of the planets with the Moon near eclipse date, using for the lunar eclipses the formulae of *Syzygies Tables*[2] and for the planets the orbits with the main perturbation terms, yielding their heliocentric positions with an accuracy of about 0°·1. Each such a conjunction thus obtained was investigated with high precision, using for the lunar positions the formulae used earlier[3], for the eclipse duration von Oppolzer[4], and for the planetary positions the tables of Tuckerman[5] and Neugebauer[6].

The same occultation types as in I were considered:

T	For some places on the Earth's surface, immersion *and* emersion of the planet by the totally eclipsed Moon;
t	For some places, immersion *or* emersion by the totally eclipsed Moon (and for *no* places immersion *and* emersion during totality);
t'	For some places, immersion and emersion at the *dark* limb during the partial phase preceding or following the totality (and nowhere *T* nor *t*, etc.);
(*t*)	The same, but at the *bright* limb;
(*t'*)	For some places, immersion *or* emersion at the *bright* limb during the partial phase preceding or following the totality;

P For some places, immersion and emersion at the *dark* limb during a partial eclipse of the Moon;
(p) The same, but at the *bright* limb;
(p') For some places, immersion *or* emersion at the *bright* limb during a partial eclipse of the Moon.

These types are illustrated in figure 2. Obviously, for T and t the planet is somewhere occulted by the totally eclipsed Moon, for P and t' by a partially eclipsed Moon on the *dark* side (independently whether the eclipse is a total or a partial one), while the symbols between brackets all represent occultations by a partially eclipsed Moon, but not by the umbra.

In Table I, the results are given for the three bright planets for the period 100 BC to AD 3000. As can be seen from this table, for some cases no positive indication about the type could be given, due to the uncertainty of the formulae and the finite apparent diameter of the planets. Since, however, these borderline cases are visible at places on Earth where the Moon is very near the horizon, it had no relevance to calculate them in more detail. For many cases, past occultations in Table I took place far from civilized areas. These cases are nevertheless of importance, since at other places they gave rise to a very close conjunction.

The Jupiter case of AD 755 is of particular interest, since this occultation has been observed and recorded by Simeon of Durham in England[7,8]. Detailed calculations show that, for Durham, immersion took place during totality, while emersion occurred during the following partial phase. Taking the difference ET − UT = +50 minutes, the following results are obtained:

maximum eclipse	$18^{h}39^{m}$	UT (from Ref. 9)
immersion of Jupiter	19 15	UT
end of totality	19 24	UT
emersion of Jupiter	20 05	UT
end of partial phase	20 23	UT

Even for a value of ET − UT as small as +35 minutes the results are similar. Jupiter was occulted by the northern part of the eclipsed Moon.

For Uranus, we limited the study to the period 1850–2050. Four cases were found, namely 1930 October 7 (type P, Eq); 1938 November 7 (type T, NH); 2014 October 8 (type T, SH); and 2022 November 8 (type T, NH). It is remarkable that the 1938 case, though visible from the east coast of North America and the Atlantic, was mentioned neither in the *Nautical Almanac* nor in the *American Ephemeris* of that year.

In addition to Table I, for the present time this investigation has been extended to penumbral eclipses. In the period 1900–2050 only two occultations of a bright planet during a penumbral eclipse have been found, both being of Saturn. The first case (1933 August 5) is of minor

TABLE I

OCCULTATIONS OF BRIGHT PLANETS BY THE ECLIPSED MOON FOR THE PERIOD −100 TO +3000

Date	*Type*	*Time of maximum eclipse (ET)*	*Magnitude of eclipse*	*Visibility*	*Remarks*
		MARS			
2 Nov. 8	P	1^h20^m	0·45	SH	
412 Nov. 4	T	21 51	1·61	SH	
916 Oct. 13	(p)	23 43	0·15	Eq	Borderline case with (*p*′)
2488 Apr. 26	T	9 42	1·39	SH	In Antarctica
		JUPITER			
103 Dec. 1	(p′)	15^h05^m	0·20	Eq	
158 June 29	(p′)	11 57	0·64	NH	Borderline case with Nil
400 Dec. 17	T	20 25	1·06	Eq	
458 Nov. 7	P	0 16	0·80	NH	
524 May 3	T	19 28	1·64	SH	
755 Nov. 23	T	19 31	1·40	NH	Observed in Europe
799 July 21	T	16 20	1·55	NH	Borderline case with *t*
810 June 20	T	20 32	1·83	Eq	Eclipse and occultation both nearly central
821 May 20	(t)	21 02	1·41	Eq	
879 Apr. 10	T	11 39	1·37	SH	
995 Jan. 19	(t′)	15 51	1·25	Eq	Borderline case with Nil
1052 Dec. 8	(t′)	22 39	1·65	NH	
1176 Apr. 25	P	19 23	0·68	SH	
1234 Mar. 17	P	3 36	0·65	SH	
1407 Nov. 15	t′	12 51	1·18	NH	
1418 Oct. 14	T	22 11	1·12	NH	Borderline case with *t*′ and Nil
1462 June 12	P	1 56	0·58	SH	Borderline case with (*p*)
1473 May 12	P	7 25	0·37	Eq	
1531 Apr. 1	(p)	18 46	0·11	SH	
2932 June 10	P	0 05	0·21	Eq	
2990 May 1	P	1 46	0·10	SH	
		SATURN			
195 July 10	t	3^h51^m	1·71	Eq	
354 Dec. 16	(t)	15 52	1·34	Eq	
502 Dec. 29	T	16 02	1·66	Eq	Borderline case with *t*
771 Feb. 4	(p)	10 40	0·94	SH	
959 June 23	P	8 34	0·94	SH	
1312 June 19	P	19 39	0·76	Eq	
1580 July 26	T	11 09	1·26	NH	
1591 Dec. 30	t	3 58	1·56	NH	
1796 Dec. 14	P	14 20	0·49	NH	
2344 July 26	T	12 42	1·33	NH	
2429 June 17	P	11 16	0·02	SH	
2829 Jan. 11	(t)	4 23	1·81	NH	
2977 Jan. 26	T	10 01	1·65	Eq	

SH, NH, Eq stand for southern hemisphere, northern hemisphere and equatorial regions, respectively.

importance since it concerns a small penumbral eclipse which was absolutely invisible without sophisticated instruments (magnitude in the penumbra 0·22). But in the second case (1944 December 29) the eclipse was a *total* penumbral eclipse with magnitude 1·02. The magnitude in the umbra was −0·02, which indicates that the Moon did not touch the umbra, but passed very near to it. Therefore the eclipse could easily be seen with the naked eye; in fact, there is little difference in the appearance of such an eclipse and a small partial one of magnitude +0·02 like, for instance, the 2429 case in Table I. The 1944 phenomenon was visible in the Pacific; at mid-eclipse the occultation could be seen near Samoa.

Finally, for the same period (1900 to 2050) there are three close conjunctions between a bright planet and the partially or totally eclipsed Moon with a geocentric separation in longitude $|\Delta\lambda|$ less than 3° at mid-eclipse: 2018 July 27 (Mars, $\Delta\lambda = +0^\circ\cdot7$), 1996 September 27 (Saturn, $\Delta\lambda = +0^\circ\cdot8$), and 2013 April 25 (Saturn, $\Delta\lambda = -2^\circ\cdot7$). Here, a positive value of $\Delta\lambda$ indicates that $\lambda_{\mathrm{Moon}} > \lambda_{\mathrm{planet}}$ so that the Moon is to the east (the 'left') of the planet at mid-eclipse. The first and the second eclipse are total ones, while the case of 2013 concerns a small partial eclipse. None of these three cases, however, gives rise to an occultation since the separation in latitude is too large. Starting from these three dates, similar close conjunctions outside this period are easily found using the periodicities of Table III described in the next section.

Frequencies and Periodicities

In figure 3, the mean frequency for various types of occultations of fixed stars by the eclipsed Moon is given as a function of its absolute latitude $|\beta|$. The graphs have been obtained by a Monte Carlo method, generating a lunar eclipse with the anomaly of Sun and Moon and the

TABLE II

CALCULATED MEAN FREQUENCIES FOR VARIOUS TYPES OF OCCULTATIONS OF SOME STARS AND PLANETS BY THE ECLIPSED MOON, EXPRESSED IN CASES PER CENTURY

	Σ	Umbra	T+t	P+t′	P	(Σ)	(p)+(p′)
Star, $\beta = 0^\circ$	1·57	1·27	0·60	0·67	0·62	0·30	0·12
α Leo, $\beta = +0^\circ\cdot46$	1·39	1·11	0·56	0·54	0·49	0·29	0·11
Mars	0·074	0·053	0·026	0·027	0·025	0·021	0·012
Jupiter	0·71	0·50	0·24	0·27	0·24	0·21	0·12
Saturn	0·39	0·27	0·13	0·14	0·13	0·12	0·07
Uranus	1·26	0·99	0·51	0·48	0·43	0·27	0·10

Σ stands for the frequency of these phenomena, independently of the type, 'Umbra' for the frequency of occultation by both the Moon and the Earth's umbra, and (Σ) for occultations by the eclipsed Moon but not by the umbra. The other symbols are described in the text.

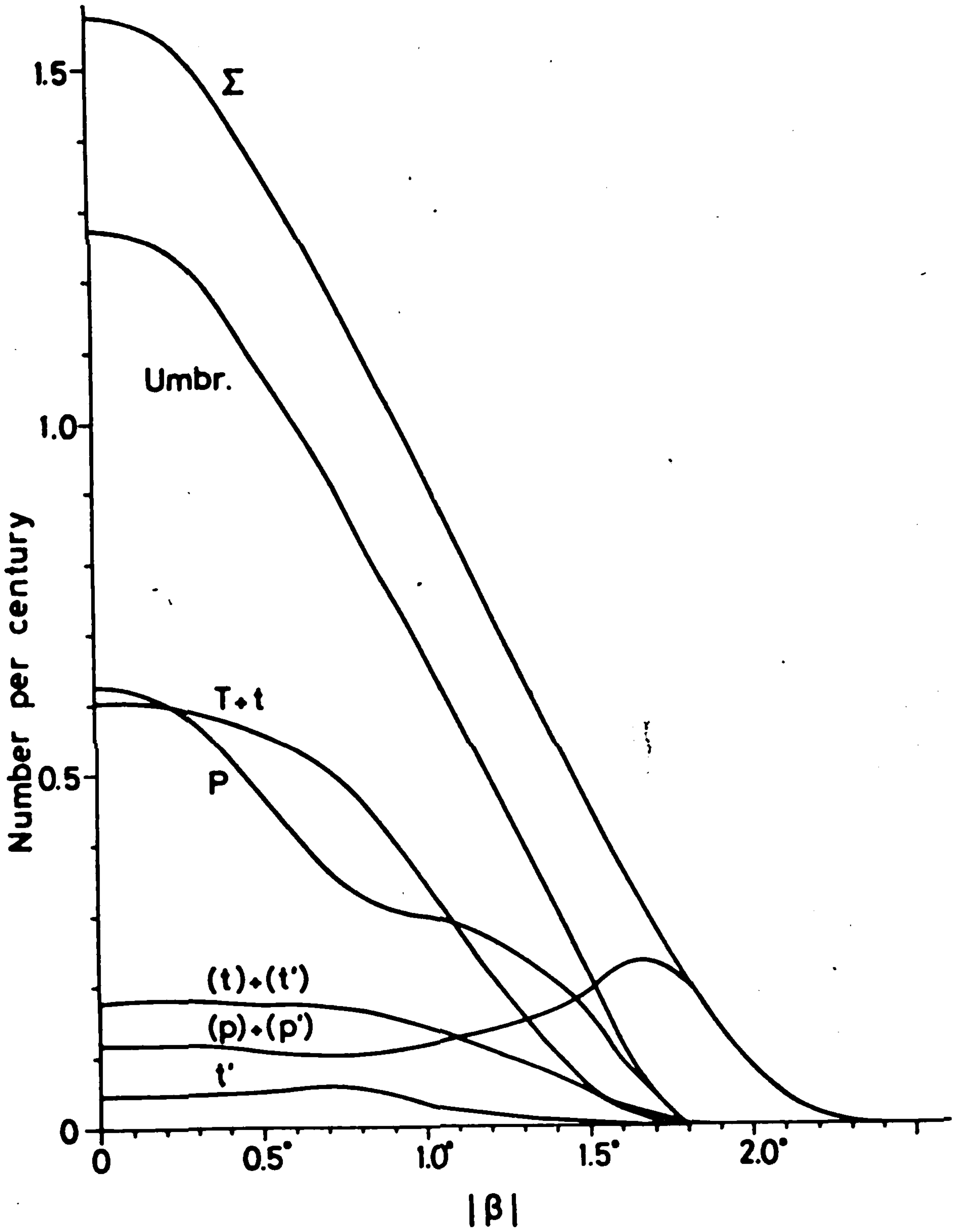

FIGURE 3. Mean frequency of various types of occultations of stars by the eclipsed Moon as a function of the absolute latitude, expressed in cases per century. Σ denotes the frequency of the phenomenon, independent of the type, and 'Umbr' stands for the frequency for occultations by both the Moon and the Earth's umbra. The symbols are described in the text.

quantity γ chosen at random and calculating the probability for the occultation types for each eclipse. (The quantity γ is the least distance of the Moon's centre to the centre of the Earth's shadow.) Integrating over the geocentric orbits of figure 1, the mean frequency for planets can be obtained, if one takes into account the retrograde motion of the planet and its synodic period. In Table II the frequencies thus obtained are given for the planets and some stars. It can be estimated that the frequency of these events for a given place is about one order of magnitude lower than for the whole Earth. Therefore, an occultation of a bright planet by the eclipsed Moon at a given place is even rarer than a total or annular solar eclipse at that place, for which the mean frequency is about 0·28 per century[10].

TABLE III

PERIODICITIES FOR CONJUNCTIONS OF THE ECLIPSED MOON WITH STARS AND PLANETS

	Years	*Lunations*	$\Delta\Omega$	$\Delta\lambda$	N_Ω	N_λ	$\Delta\Omega_{\text{planet}}$
Star	19·00	235	+ 7°·57	−0°·18	*2·9*	25·0	—
	65·01	804	− 0°·91	+0°·92	24·5	*4·9*	—
	781·03	9660	− 2°·91	+0°·27	*7·7*	17·0	—
Mars	46·98	581	− 0°·44	−1°·78	51·3	*2·5*	− 8°·0
	192·19	2377	+ 3°·79	−0°·05	*5·9*	84·0	+ 65°·2
	4529·17	56018	+ 0°·41	+0°·06	*54·0*	75·2	+ 26°·4
Jupiter	10·92	135	+ 0°·52	−2°·47	43·1	*1·8*	− 28°·5
	68·81	851	+ 0°·60	+0°·93	37·2	*4·8*	− 71°·7
	3357·14	41522	+ 0°·76	−0°·51	29·6	*8·8*	+ 16°·8
Saturn	68·32	845	− 3°·42	−0°·76	6·5	*5·9*	+116°·5
	131·47	1626	+10°·24	−0°·99	*2·2*	4·5	+167°·3
	484·47	5992	− 2°·33	+0°·29	*9·6*	15·2	+171°·5
Uranus	84·01	1039	+ 6°·66	+0°·82	*3·4*	5·4	+ 0°·7
	144·73	1790	+ 0°·21	−1°·14	108·5	*3·9*	− 98°·6
	426·09	5270	− 6°·43	+0°·88	*3·4*	5·1	+ 29°·5

$\Delta\Omega$ is the increase of $(\lambda_{\text{Moon}} - \lambda_{\text{lunar node}})$, $\Delta\lambda$ the increase of $(\lambda_{\text{Moon}} - \lambda_{\text{planet}})$, and $\Delta\Omega_{\text{planet}}$ the increase of $(\lambda_{\text{planet}} - \lambda_{\text{acs. node planet}})$ after such a time interval. $N\Omega$ is the mean length of a sequence of lunar eclipses for such a periodicity, and $N\lambda$ the length of a series of conjunctions with $|\lambda_{\text{Moon}} - \lambda_{\text{planet}}| < 2^\circ{\cdot}25$. The shortest one of these lifetimes is printed in italics.

As can be seen from I (Table III, Regulus) and from Table I, the mean frequencies are in agreement with the calculated ones in Table II. On the other hand, the actual frequencies of, for instance, Jupiter or Regulus clearly show fluctuations: the frequency is relatively high for a long period and zero for a subsequent period. The same holds for close conjunctions with the eclipsed Moon, which is obviously a necessary condition for an occultation. The reason for this is that the distribution in time of the event is determined by so-called periodicities. For a planet or star on the ecliptic such a periodicity is a time interval in which the number of synodic revolutions of the Moon (lunations) is an integer, while i, half its number of draconic revolutions, and ii, the number of synodic revolutions of the star or the planet, are almost integers. These periodicities are thus characterized by two numbers, one representing the displacement of the Full Moon with respect to its node, and the other its geocentric displacement with respect to the star or planet. A 'good' periodicity is one whose lifetime is long, i.e. one which produces a long series of events. Such a series ends when either the lunar displacement with respect to the node becomes so large (>22°·4) that there will be no eclipse any more, or when the displacement with respect to the star or planet becomes so large (>4°·5) that nowhere on Earth an occultation can take place during eclipse. In principle, such 'good' periodicities can be found by the use of

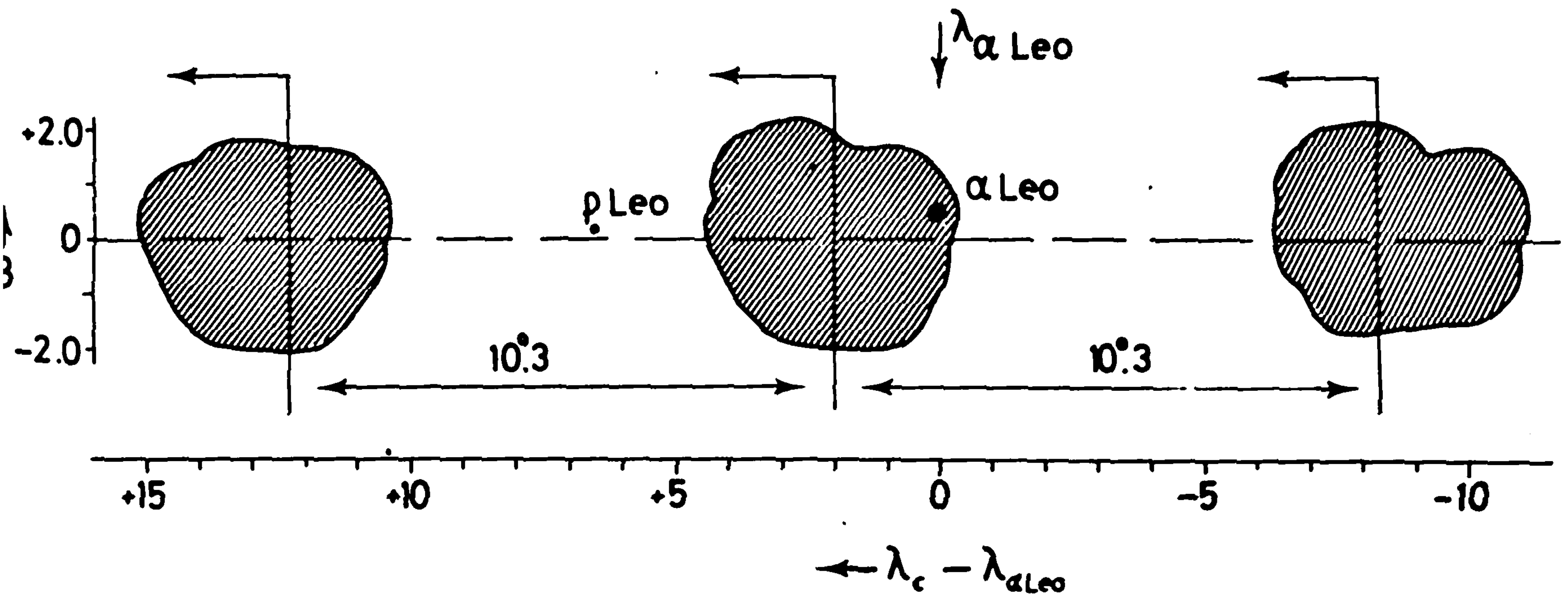

FIGURE 4. Places on the ecliptic near α Leo where occultations by the eclipsed Moon take place during the period AD 1900–2030.

continued fractions of higher order[11]. In our case, however, a computer method is more convenient, since it may take into account the difference in weight of conditions i and ii. In Table III the periodicities thus obtained are given together with the displacements and the maximum lifetimes. The last column in Table III gives the displacement $\Delta\Omega_{\text{planet}}$ with respect to the node of the planetary orbit. It is to be noted that the periodicities are strictly valid for *conjunctions* and not for occultations, and that they are calculated assuming *circular* orbits. However, if (as for Jupiter or Uranus) the inclination of the orbit is small, they may also be used for occultations. But for Mars, and also for Saturn, an occultation may only be expected if $\Delta\Omega_{\text{planet}}$ is within certain limits, as can be seen from figure 1. Also, the ellipticity of the orbits will give rise to deviations, since the velocity of the planet is not uniform over the orbit.

Therefore, the displacement of the planet with respect to the eclipsed Moon may deviate from Table III if the number of anomalistic revolutions of the planet is not an integer. For most planets this effect is of minor importance; for Jupiter its effect will be discussed at the end of this section. But, again, for the close planet Mars this effect is so pronounced that it is no use selecting periodicities for which its number of anomalistic revolutions is not almost an integer. Therefore, for this planet only such periodicities are listed in Table III.

The effect of the periodicities on the distribution in time of the event is best illustrated for stars. In figure 4, the areas are given near Regulus where stars are occulted by the eclipsed Moon for the period 1900–2030. Clearly, these occultations take place only in small areas of the ecliptic. These areas are approximately circles of 4° diameter. Over the whole ecliptic there exist 35 of these areas, their mutual distance thus being 10°·3. In each area, lunar eclipses repeat themselves with intervals of 19 and 65 years, hence the short periodicities of Table III. Such a series of lunar eclipses is infinite, since the lunar nodal displacement of both periodicities may always compensate each other. Every time, however, when the 65-year period is involved, there is a displacement $\Delta\lambda$ of the eclipsed Moon with respect to the stars; this displacement cannot be compensated by the (small) displacement of the 19-year period in such a sequence of lunar eclipses. As a consequence there is a slow direct motion of the areas. After a period of about 781 years the next area has arrived at the place of the first one. From this it is clear that for these phenomena *three* periodicities are of relevance; they have been listed in Table III. Obviously, if the lunar nodal displacement of both first periodicities can compensate each other while also the shift with respect to the stars (or, more generally, to an object) is small, the movement of the areas is slow and thus the third periodicity is long. In that case the frequency of the events is relatively high during a long period, which is followed by a somewhat longer period in which not even one occultation or close conjunction is produced during eclipse. This is the case for stars or, even more striking, for Jupiter. Contrarily, for a planet like Saturn or Uranus no such a long periodicity plays a role, and therefore the occultations and close conjunctions during eclipse are more or less uniformly distributed in time.

For stars, the short periodicities in Table III have a different character. The first one has a large nodal displacement but a small shift with respect to the star, while for the 65-year period it is just opposite, as can be seen from the lifetimes N_λ and N_Ω in Table III. For Jupiter, however, both periodicities have a small displacement with respect to the lunar node, thus producing long sequences of lunar eclipses. As a consequence, for stars there are occultations at which both the partially and the totally eclipsed Moon is involved during the whole period at which the occultations take place, due to the large nodal displacement of the 19-year period.

For Jupiter, on the other hand, no such a periodicity is involved, so that there is only a slow nodal displacement in a series of occultations. Therefore, a period of occultations starts and ends with occultations by the partially eclipsed Moon, while the totally eclipsed Moon may occult the planet only in the middle of the period, as can be seen from Table I. Therefore, the time interval between two series of occultations by the *totally* eclipsed Moon is very long in this case: the next one will not occur before AD 3500, while the event preceding the AD 400 case was in the twentieth century BC.

The influence of the eccentricity of the planetary orbits can be seen from the Jupiter results in Table I. For circular orbits, periodicities of 10·92 and 68·81 years are to be expected. The former one corresponds to 0·92 revolution of Jupiter, so that after such a period the planet has almost completed one revolution. If the remaining part of 0·08 orbit is situated near perihelion, the velocity of Jupiter in this part will be above its mean value. Therefore, after 10·92 years the longitude of Jupiter (λ_{Jup}) will be smaller than expected from a circular orbit, which results in a larger $\Delta\lambda = \lambda_{\text{Moon}} - \lambda_{\text{Jup}}$ in this case as given in Table III. Near perihelion $\Delta\lambda$ may reach a maximum value of $-2^\circ{\cdot}47+3^\circ{\cdot}41 = +1^\circ{\cdot}06$, so that here this periodicity is much better than should be inferred from Table III. Indeed, near perihelion this periodicity dominates, producing long, though sometimes interrupted, occultation series (103–158; 458–524; 755–799–810–821; 1407–1418–1462–1473). Such a series ends when Jupiter is too far from perihelion and $\Delta\lambda$ becomes too small.

For the 68·81-year periodicity, in principle the same argument holds. But in this case, the deviation of $\Delta\lambda$ from its mean value may become very high (up to 8°·2) due to the larger 'missing part' of the orbit. Therefore, only in a small area of the orbit this periodicity satisfies; in Table I there is only one example (1462–1531). On the other hand, the periodicity of 57·89 years (68·81 minus 10·92), which does not appear in Table I, turns out to be very favourable for describing the phenomenon near aphelion. For this periodicity, $\Delta\lambda = +3^\circ{\cdot}74$ and $\Delta\Omega = +0^\circ{\cdot}08$. If the 'missing part' of 0·11 orbit is situated near aphelion, a part of the orbit is not travelled by Jupiter where its velocity is low. Therefore after such a period the longitude of Jupiter is larger than expected for a circular orbit. Thus $\Delta\lambda$ is smaller than given in Table III and may reach a minimum value of $+3^\circ{\cdot}74-5^\circ{\cdot}03 = -1^\circ{\cdot}29$. Therefore, at aphelion it is the 57·89-year periodicity which governs the distribution in time of the occultations, while the 10·92-year periodicity does not play any role. Examples in Table I are 400–458; 524–755; 821–879–995–1052; 1176–1234–1407 and 1473–1531. Again, such a series ends when Jupiter has drifted so far from aphelion that $\Delta\lambda$ has become too large. At this point, however, the 10·92-year periodicity has become good enough to overtake its role.

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