SMJK KATHOLIK,PJ
2010 STPM Trial Examination
Mathematics T/S
Paper 1

Upper Six
Prepared by: Mdm. Tan CT

Time: 3 hours
Total: 100%

Instructions to candidates:
Answer **ALL** questions.
All necessary working should be shown clearly.
Non-exact numerical answers may be given correct to three significant figures, or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

1. Simply $\dfrac{(r+\frac{1}{s})^r(r-\frac{1}{s})^s}{(s+\frac{1}{r})^r(s-\frac{1}{r})^s}$. [4]

2. Given that $(x-yi)^2 = 5-12i$, where $i=\sqrt{-1}$. Find all the real values of x and y . [5]

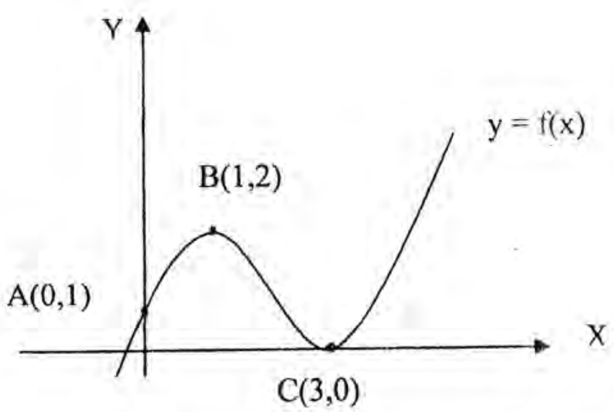
3. Show that $kb^2 = (1+k)^2 ac$, if the ratio of the roots for the equation $ax^2+bx+c=0$

   is $k:1$. [5]

4. Show that $\dfrac{d}{dx}(\tan^5 x) = 5\sec^6 x - 10\sec^4 x + 5\sec^2 x$. [3]

   Hence, find the indefinite integral $\int(\sec^6 x - 2\sec^4 x)dx$. [3]

5.



In the diagram above, the curve y = f(x) passes through the points A(0 , 1), B (1 , 2) and C (3 , 0). On the separate diagrams, sketch the graphs of :

(a) y = f(x + 2) [2]
(b) y = f(2x) [2]
(c) y = − 2f(x) [2]

6. Find the sum of the first n terms of the series.

$$\frac{1}{3(4)}+\frac{1}{4(5)}+\frac{1}{5(6)}+\frac{1}{6(7)}+\ldots\ldots$$ [6]

7. Determine the set values of x such that satisfy the inequality $\frac{|x-2|+1}{|x-2|-1}<3$. [7]

8. With the help of a sketch graph, show that the equation $\sin(x+\frac{\pi}{3})=\tan x$ has only one positive root in the range $0 \le x \le \frac{\pi}{2}$.

   Show that this root lies between $\frac{\pi}{6}$ and $\frac{\pi}{4}$. [3]

   Use the Newton-Raphson method, and take 0.7 as the first approximation, determine two further approximation roots of the above equation correct to 3 decimal places. [5]

9. (a) Given that $A=\begin{pmatrix}0 & -2\\ 2 & 0\end{pmatrix}$, find

   i. A^2.
   ii. A^4.
   iii. A^{23}. [6]

   (b) Calculate the possible values of k, given that

$$\begin{vmatrix}2-k & 2 & -1\\ 2 & 4 & -2\\ 3 & k+2 & k-3\end{vmatrix}=0.$$ [4]

10. Prove that the equation of the tangent to the rectangular hyperbola $xy=c^2$ at the point $P(cp,\frac{c}{p})$ is $x+p^2y=2cp$. [4]

   If the tangents at the points $P(cp,\frac{c}{p})$ and $Q(cq,\frac{c}{q})$ intersects each other at the point T(X , Y), prove that $pq=\frac{X}{Y}$ and $p+q=\frac{2c}{Y}$. [6]

   If the chord PQ has a fixed length of d, show that $d^2=c^2(p-q)^2(1+\frac{1}{p^2q^2})$. [3]

11. Given a curve $y = \dfrac{8x-10}{x^2-1}$.

(a) State the equations of the asymptotes of the curve. [2]

(b) Find the points on the x-axis and y-axis intersected by the curve, and all the stationary points of the curve. [6]

(c) Sketch the curve $y = \dfrac{8x-10}{x^2-1}$. [4]

(d) On the separate diagram from (c), sketch the curve that has the equation

$$y = \frac{x^2-1}{8x-10}.$$ [3]

12. (a) The function f is defined by

$$f(x) = \begin{cases} x^3+1 & ,x<2 \\ 5+4x-x^2 & ,x \geq 2 \end{cases}$$

Sketch the graph of f. [3]

Calculate the area bounded by the curve and the x-axis. [3]

(b) The functions f and g are defined as:

$$f(x) = \frac{x+|x|}{2} \quad , \quad x \in R$$

$$g(x) = \begin{cases} x-1 & ,x<0 \\ 2 & ,x>0 \end{cases}$$

Sketch the graphs of the function f and g. [5]

Without using graphs, show that the composite function $f \circ g$ is not continuous at $x = 0$. [4]

END OF QUESTION PAPER

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Paper 2

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1. Express $3\sin\theta+\frac{1}{2}\cos\theta$ in the form $R\sin(\theta+\alpha)$, where $R>0$ and $0^{\circ}<\alpha<90^{\circ}$.

   Hence, find the maximum and minimum values of $\dfrac{1}{3\sin\theta+\frac{1}{2}cox\theta}$. [5]

2. An observer at a point O on a horizontal plane observes two other points, P and Q.
   The point P is on the horizontal plane containing O on a bearing Nα°W from O.
   The point Q is situated due north of O at an angle of elevation β as observed from O.
   If OP = OQ = r, show that the length l of the straight line PQ is given by
   $$l^2=2r^2(1-\cos\alpha\cos\beta).$$
   Hence, or otherwise, show that $\cos\angle POQ=\cos\alpha\cos\beta$. [7]

3. A ball bearing is dropped from point O at the surface of a viscous liquid in a tall cylinder.
   The speed of the ball bearing is accelerated by gravitational acceleration g ms^{-2} but is retarded by the resistance of the liquid at a magnitude of kx ms^{-2}, where x m is the displacement of the ball bearing from the surface of the liquid and k is a positive constant.

   Show that $v\dfrac{dv}{dx}=g-kx$. [1]

   Hence, show that

   (a) $v^2=2gx-kx^2$. [3]

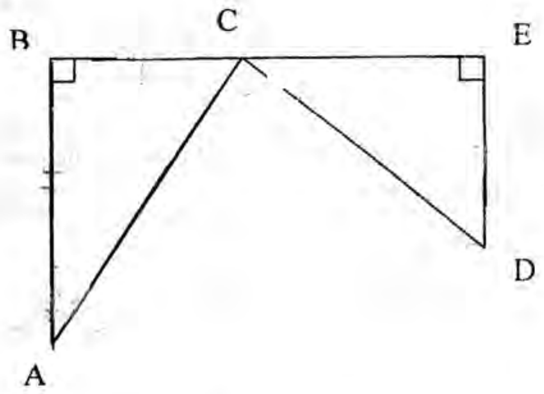
   (b) the terminal velocity of the ball bearing is given by $\dfrac{g}{\sqrt{k}}\ ms^{-1}$. [3]

4. ABCD is a quadrilateral. P is the point of intersection of the diagonals AC and BD such that $\overrightarrow{AP}=\lambda\overrightarrow{PC}$ and $\overrightarrow{BP}=\mu\overrightarrow{PD}$, where λ and μ are constants.

   Show that $\overrightarrow{OP}=\dfrac{1}{1+\lambda}\overrightarrow{OA}+\dfrac{\lambda}{1+\lambda}\overrightarrow{OC}$ and $\overrightarrow{OP}=\dfrac{1}{1+\mu}\overrightarrow{OB}+\dfrac{\mu}{1+\mu}\overrightarrow{OD}$, where O is the origin. [4]

   If the position vectors of A, B, C and D, relative to O, are $-\underline{i}+\underline{j}$, $2\underline{i}+4\underline{j}$, $2\underline{i}+9\underline{j}$ and $-2\underline{i}+8\underline{j}$ respectively, determine the values of λ and μ . [5]

5. The diagram below shows two congruent triangles ABC and CED, with AB = CE, BC = ED and $\angle B = \angle E = 90^\circ$. BCE is a straight line.



Show that: (a) $\angle ACD = 90^\circ$ [3]

(b) ACD is an isosceles triangle. [2]

If BE = 6 cm and AB : BC = 2 : 1, find

(c) the area of trapezium ABED [3]

(d) the are of triangle ACD [2]

6. Two boats A and B travel with constant speeds of $4\sqrt{2}$ kmh^{-1} and 5 kmh^{-1} in the direction north-east and S θ°E respectively. Given that $\theta = \tan^{-1}\left(\frac{3}{4}\right)$, find the magnitude and direction of the velocity of A relative to the velocity of B. At 12.00 noon, B is 80 km north of A. [4]

(a) Determine the closest distance between these two boats and the distance traveled by each boat to reach their respective positions then. [5]

(b) If the pilot of A intends to intercept the boat B, find the direction that he must now steer A in order to intercept B. Hence, find the time at the instant when A intercepts B. [4]

7. The numbers of days taken for a sick leave by factory workers is normally distributed with a mean of 12 days and variance σ^2 in a year. If the probability of less than 10 days off is 0.1587, find the variance σ^2. Hence, find a if $P[\,|x-12| < a\,] = 0.950$. [4]

8. 7 out of 20 visitors to a Travel Fair are expected to purchase air tickets. Find the probability that at least two out of three visitors chosen at random would purchase air tickets. [2]

20000 visitors are expected for the second day of the Travel Fair. By using an approximation method, calculate the probability that not more than 7050 of these visitors would purchase air tickets. [4]

9. Five red balls and three blue balls are placed in a bag. The balls are indistinguishable from one another apart from their colours. Three balls are drawn without replacement in the bag. The variable X represents the number of blue balls drawn.

Show that $P[X = 0] = \frac{5}{28}$ and hence, form a probability distribution table for the number of blue balls drawn. [5]

Calculate the mean and standard deviation of X. [3]

10. A total of 400 new students at a college were interviewed to find out if they either received a scholarship, loan or no financial aid. There are 150 male students, of which 50 receive loan and 70 do not receive any financial aid. One hundred female students receive scholarship. There are 140 students who do not receive any financial aid. If a new student is selected at random, calculate the probability that the student is a

(a) female or a scholarship recipient. [2]
(b) loan recipient if it is known that the student is female. [2]
(c) male who is a scholarship recipient or a female who receives loan. [2]
(d) female or non-scholarship recipient. [2]

11. The continuous random variable X has the probability density function,

$$f(x) = \begin{cases} k(4x - x^2) & 0 < x < 2 \\ 0 & otherwise \end{cases}.$$

(a) Find the value of k. [2]

(b) Calculate $P[X \leq \frac{1}{2}]$. [2]

(c) Calculate Var(X) and Var (2X − 1). [6]

12. The marks scored by 25 students in a Physics test are as given below:

30	57	89	42	62	45	52	40	78	48	32	64	70
24	83	65	23	38	55	52	76	54	65	38	48	

(a) Display the above data in an ordered stem plot. [2]
(b) Find the mean, median and interquartile range. [6]
(c) Draw a box plot to represent the data. [3]
(d) State the shape of the frequency distribution, giving a reason for your answer. [2]