Firdaus

SULIT

Mathematics T
September 2010
3 hours

NAME:

FORM U6: A | B

SMK ST JOSEPH KUCHING

TRIAL EXAMINATION 2010

UPPER 6

MATHEMATICS T

PAPER 1 (954/1)

3 hours

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

Answer **all** *questions..*

All necessary working should be shown clearly.

Non –exact numerical answers may be given correct to three significant figures, or One decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Total marks:	

This paper consists of 3 printed pages.

*Answer **all** questions.*

1. Using the laws of the algebra of sets, show that, for any sets A and B,
$(B-A)\cup(A-B)=(A\cup B)-(A\cap B)$ [5]

2. If $y-2e^{-2x}\sin x=0$, prove that $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5y=0$. [4]

3. If $f(x)=\frac{2}{(x-3)(1+x)}$, express $f(x)$ in partial fractions. Given that x is sufficiently small for x^4 and high terms to be negligible, show that $f(x)\approx-\frac{2}{3}+\frac{4}{9}x-\frac{14}{27}x^2+\frac{40}{81}x^3$. Determine the set of values of x for which this expansion is valid. [12]

4. a) A triangle has vertices $A(0,2)$, $B(3,5)$ and $C(5,-1)$. Find
i) the coordinate of the point M on AB such that $MB=2\,AM$,
ii) the perpendicular distance from B to AC. [6]

b)Find the centre and radius of each of the circles $x^2+y^2-4x-2y-20=0$ and $x^2+y^2-16x-18y+120=0$. Hence deduce that the two circles touch each other externally and write down their common tangent at the point of contact. [9]

5. Sketch the graphs for $y=\frac{2}{|x|}$ and $2y=3|x|-4$ on the same axes. Hence, find the range of values of x for which $\frac{4}{|x|}<3|x|-4$. [6]

6. For the geometric series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots$, calculate the least value of n so that the difference between the sum of the first n terms and the sum to infinity is less than 10^{-7}. [5]

7. The function f is defined by $f:x\to 1+2\sqrt{x+2}$ for $x\geq -2$.
a) Find $f^{-1}(x)$ and state the domain of f^{-1}. [3]
b) Sketch the graph of f and the graph of f^{-1} on the same axes. State the line of symmetry. Hence or otherwise find the value of x for which $f(x)=f^{-1}(x)$. [6]

8. Given that $z_1=1+2i$ and $z_2=3-4i$. Express $\frac{1}{z_1}+\frac{1}{z_1-2z_2}$ in the form $a+bi$, where a and b are real numbers. Represent this number in an Argand diagram. Hence find it modulus and argument. [7]

9. Given $A = \begin{pmatrix} 4 & 2 & 1 \\ 9 & 3 & 1 \\ 16 & 4 & 1 \end{pmatrix}$ find A^{-1}.

The graph of a quadratic equation $y = ax^2 + bx + c$ through the points whose coordinates are (2,1), (3,0) and (4,2).

i) Obtain a system of linear equations to represent the given information.

ii)Rewrite the system of linear equations as a matrix equation. Hence, find the quadratic equation of the graph [10]

10. Sketch on the same coordinates axes, the graphs of $y = e^{-x}$ and $y = \frac{2}{1-x}$.

If the x-coordinate of the point of intersection is p. Show that $-1 < p < 0$.

Use the Newton-Raphson method to approximate the value of p correct to two decimal places. Hence find the point of intersection. [12]

11. A region is bounded by the curve $y = x^2$, the line $x = 1$, the y-axis and the curve $y = e^{x+1}$. Prove that the area of this region is $e^2 - e - \frac{1}{3}$. [5]

12. A region bounded by the curve $y = \sqrt{x-1}$, the axes and the line $y = 2$. Find the volume of the solid generated when this region is rotated through 2π radians.

a) about the x-axis b) about the y-axis. [10]

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MATHEMATICS T

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Total marks:	

This paper consists of 4 printed pages.

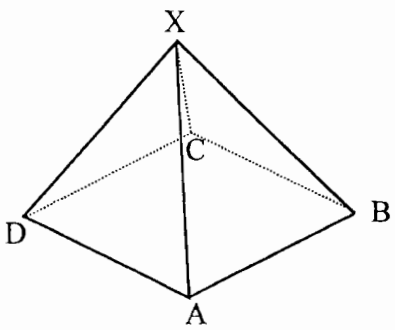
*Answer **all** questions.*

1. Show that the differential equation $2x^2 \frac{dy}{dx} = xy + y^2$, where $x > 0$, can be reduced with the of the substitution $y = u\sqrt{x}$, into the form $2x^{\frac{3}{2}} \frac{du}{dx} = u^2$. Hence, find the general solution by expressing y in terms of x. [7]

2. Solve the differential equation $\frac{dy}{dx} = \frac{y^2}{(x^2 - x - 2)}$ which satisfies $y = 1$ when $x = 5$ in region $x > 2$. [10]

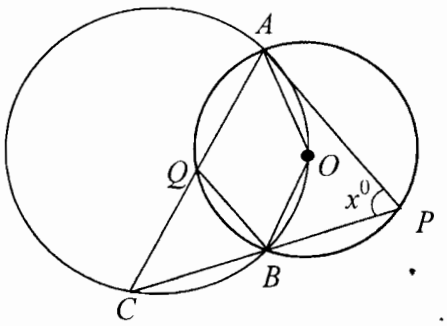
3. In a child's game there should be 7 triangles, 3 of which are blue and 4 of which are red, ~~11 squares, 5 of which are blue and 6 of which are red~~ and 11 squares, 5 of which are blue and 6 of which are red. However, two pieces are lost. Assuming the pieces are lost at random, find the probability that they are
i) the same shape ii)the same colour iii) the same shape and colour
iv) the same shape given that they are same colour. [8]

4.



The figure shows a solid where ABCD is a rectangle which is horizontal. Given that AB=13cm, BC = 8cm, AX=DX=9cm and BX=CX=7cm, calculate
a) the area of triangle AXB,
b) the perpendicular distance from X to AB,
c) the angle of inclination of the face AXB to the horizontal, hence find the height of X above the base ABCD. [7]

5. In the diagram , AQC and PBC are straight line lines and O is the centre of circle of circle AQBP.

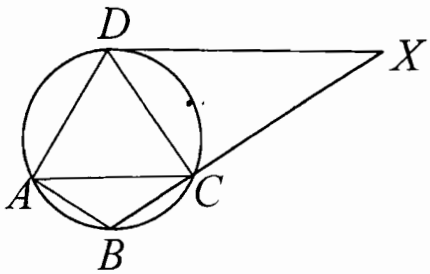


Given $\angle APB = x^0$, write down $\angle AOB$ and $\angle ACB$ in term of x. Prove that

a) ΔCQB is an isosceles triangle.

b) QB is parallel to AP. [8]

6.



In the diagram, $ABCD$ is a cyclic quadrilateral. A straight line drawn from D parallel to AC meets BC produced at X. Prove that triangles ABD and CDX are similar. Given $CD = 4.2$cm, $DA = 5.6$cm and $CX = 2.1$ cm, calculate the length of AB. [6]

7. Given that $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$, $\overrightarrow{OP} = \frac{4}{5}\overrightarrow{OA}$ and that Q is the midpoint of AB, express $\overrightarrow{AB}$ and $\overrightarrow{PQ}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. PQ is produced to meet OB produced at R, so that $\overrightarrow{QR} = n\overrightarrow{PQ}$ and $\overrightarrow{BR} = k\underset{\sim}{b}$. Express $\overrightarrow{QR}$

i) in terms of n, $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

ii) in term of k, $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

Hence find the value of n and of k. [7]

8. In a school , 30% of the students cycle to school.

a) From a sample of 5 students, calculate the probability that at least 4 students cycle to school.

b) Find the minimum number of students that have to be chosen so that the probability that at least one student cycles to school is more than 0.9. [7]

9. A discrete random variable X has the following probability distribution.

x	1	2	3	4
$P(X=x)$	c	$c+2c^2$	$c+c^2$	c^2

Where c is a constant.

a) Find the value of c.

b) Find the mean and variance of X. [6]

10. If $X \sim N(20,25)$ find

a) $P(|X-20|<10)$ [4]

b) $P(X>a)=0.572$ [4]

11. The probability density function of a random variable X is given by

$$f(x)=\begin{cases}\dfrac{x}{300} & 0\le x\le 20\\ \dfrac{1}{5}-\dfrac{x}{150} & 20\le x\le 30\\ 0 & \textit{otherwise}\end{cases}$$

a) Sketch the graph of $f(x)$.

b) Find the expected value $E(X)$.

c) Find the value of k that satisfies $P(k\le X\le 25)=\dfrac{13}{24}$. [10]

12. The grouped frequency distribution table gives the lengths, recorded to the nearest mm, of 250 nails produced from a particular machine.

Length (mm)	98-99	100-101	102-103	104-105	106-107	108-109
Number of nails	12	36	106	58	30	8

a) Calculate the mean and standard deviation . [5]

b)Draw the cumulative frequency curve and from your graph , estimate

i) the interquartile range,

ii) the percentage of nails having a length below 103 mm. [9]

Given that the top 10% and bottom 10% of this distribution are rejected, estimate from your graph the limits between which the length of a nail must lie if it is to be accepted. [2]